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Exploiting Diversity in Macroeconomic Modeling:
a comparative study between Agent-Based and DSGE Macroeconomic Models

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Dissertação de Mestrado, apresentada ao Programa de Pós-Graduação em Economia da Faculdade de Ciências e Letras – Unesp/Araraquara, como requisito para obtenção do título de Mestre em Economia.

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A todos que acreditam em meus planos e incentivam essa jornada. A Deus, por uma segunda chance.

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A Deus. Aos meus pais. À minha família. Ao meu orientador. Aos verdadeiros amigos.

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“The difficulty lies, not in the new ideas, but in escaping from the old ones […]”

John Maynard Keynes (1936, Preface, p. xvii)

ABSTRACT

Macroeconomic modelling has been under intense scrutiny since the Financial Crisis of 2007-2008, when serious shortcomings were exposed in the DSGE methodology. Although many of these criticisms were unfair or uninformed, they did highlight the need of considering alternative forms of macroeconomic modelling and enhancing established approaches in order to make them more useful for understanding a world in recession. In this sense, we argue that exploiting diversity in macroeconomic modelling can benefit the profession and yield more fruitful developments regarding the formulation of macroeconomic policy. One way of exploring diversity in macroeconomics is by investigating systematically both the DSGE and the Agent-Based models, revealing their relative strengths and limitations, and combining these two different approaches, so that we can explore what one can learn from the other and perhaps yield a hybrid model. This work takes the first step towards this ultimate achievement. We believe that an interdisciplinary approach may help not only the entire macroeconomic research agenda, but also benefit society as a whole, allowing the implementation of more effective policy measures and by increasing the ability of economists to model social heterogeneity in a complex-evolving world.

KEY WORDS: Macroeconomic Policy. New Neoclassical Synthesis. New Keynesian Models. DSGE Models. Agent-Based Computational Economics. Agent-Based Models. Complexity Theory.

RESUMO

A modelagem macroeconômica tem estado sob intenso escrutínio desde a Crise Financeira de 2007-2008, quando graves deficiências foram expostas na metodologia DSGE. Embora muitas dessas críticas tenham sido injustas ou desinformadas, elas enfatizaram a necessidade de considerar formas alternativas de modelagem macroeconômica e aprimorar abordagens estabelecidas, a fim de torná-las mais úteis para a compreensão de um mundo em recessão. Nesse sentido, argumentamos que explorar a diversidade na modelagem macroeconômica pode beneficiar a profissão e produzir resultados importantes em relação à formulação de políticas macroeconômicas. Uma maneira de explorar a diversidade na macroeconomia é investigar sistematicamente tanto os modelos DSGE quanto os modelos baseados em agentes, revelando suas forças e limitações relativas, e combinando essas duas abordagens diferentes, a fim de que possamos aprender uma com a outra e talvez produzir um modelo híbrido. Este trabalho dá o primeiro passo rumo a esse desafio. Acredita-se que uma abordagem interdisciplinar pode ajudar não só toda a agenda da pesquisa macroeconômica, mas também beneficiar a sociedade como um todo, permitindo a implementação de medidas políticas mais eficazes e aumentando a capacidade dos economistas em modelar a heterogeneidade social em um mundo complexo e em constante evolução.

PALAVRAS-CHAVE: Política Macroeconômica, Síntese Novo-Neoclássica, Modelos Novos-Keynesianos, Modelos DSGE, Economia Computacional Baseada em Agentes, Modelos Agent-Based.

SUMMARY

1. INTRODUCTION

The clash between two competing business cycle theories – the Real Business Cycle (RBC) perspective and the New Keynesian paradigm – ended in the last decades with the development of a New Neoclassical Synthesis, grounded upon Dynamic Stochastic General Equilibrium (DSGE) models, which are at the heart of contemporary macroeconomic theory (Fagiolo and Roventini, 2012, 2017). Such models derive from the RBC models whose methodological influence is enormous so that it is worthwhile to make some remarks.

Under this approach, the model is not the result of research, but a tool for inferring policy implications. Such models use microeconomic fundamentals as representative optimizing agents and perfect competition to respond to macroeconomic issues. Generally, the aggregation behavior is the sum of individual behavior, in the sense that the field of macroeconomics corresponds essentially to the problem of aggregation. In this view, macroeconomics may be thought of as taking individual utility or profit maximizing behavior and translating it to the aggregate level (Kirman, 2006).

The process of such work can be broadly described as follows. Once the researcher has a quantitative query, he uses theories that have undergone rigorous empirical tests to build his model. Then he calibrates the model using real long-term data and uses that model to answer his initial query. Such a research procedure is called by its authors as a computational experiment and its authors would be creators of quantitative laboratories. The business cycles for the authors of RBC models originate from exogenous technological shocks and are propagated through the economy by agents smoothing consumption and optimizing between leisure and consumption over time.

On the other hand, the new Keynesian (NK) models have rational expectations of all agents with price stickiness. They contemplate three equations: an equation of the Keynesian Philips curve, an equation of the IS curve - which assumes the permanent income hypothesis - and a monetary rule equation that incorporates the behavior of the central bank with inflation targets.

In RBC models, flexible price equilibrium is also the first best. It is not possible to achieve higher welfare than at this equilibrium. However, in the NK-model, flexible price equilibrium is not the first best because exist imperfect competition in the goods market. Thus, welfare can be improved if this imperfection is removed.

Note that when the economy is disturbed by some shock, the cycles that emerge in the RBC models are equilibrium cycles. Moreover, they are efficient because they represent the first best outcomes and there is no role for the government to act with the goal of improving welfare. In the NK-model, price stickiness keeps the economy out of flexible price equilibrium so that welfare can be improved by some public policy intervention. Still, such cycles are cycles of equilibrium because they originate from forward-looking agents that always have a best-response decision. The mechanism that generates business cycles for the NK authors is the intertemporal substitution of labor, so that there is no involuntary unemployment. In addition, all agents have rational expectations, which lead, for instance, to inflation jumping from one level to another.

One of the criticisms that can be made about DSGE models is the use of representative agent simplification. The Walrasian economy (Colander, 2006) uses as an element of the economic system the utility maximizing agent considering that its behavior prevails in the aggregate, assuming that the choice of representative agent will coincide with the aggregate choice of individuals in society. As stated by Lengnick (2013), this is considered by a great number of economists a way to provide proper microfoundation[1]. However, it seems clear that the representative agent's reaction to a policy change will not necessarily be the same as that of individuals in society and it also seems obvious that the preferences of the representative agent may be radically different from those of society as a whole. According to Kirman (1992), individual optimization does not imply collective rationality. As he argues, "there is no direct relation between individual and collective behavior" (Kirman, 1992, p. 118).

Forni and Lipi (1997, 1999) show that the basic properties of linear dynamic microeconomic models are not maintained in the aggregate if the agents are heterogeneous. For example, a micro-level cointegration does not lead to macro cointegration, just as Granger's causality test may not happen at the micro level but occur at the macro level. In general, the representative agent hypothesis implies a one-to-one

[1] According to the author, “in most macroeconomic models (…) microfoundation is either obtained by setting the aggregate equal to a ‘representative’ individual or by summing up over all individual decisions and confronting these sums on an aggregate level (the market). As a result, phenomena of the macro level are directly linked to individual behavior”. (Lengnick, 2013, p. 102).

correspondence between the micro and macro levels. This simplification prevents DSGE models from accounting for complex events.

Some empirical evidences have shown that the non-linearity of the economy may lead to different consequences of macroeconomic policy depending on the state of the economy (e.g. Auerbach and Gorodnichenko, 2012) and the financial market (Mittnik and Semmler, 2013; Ferraresi et al., 2014). In DSGE models, the impacts of monetary and fiscal policies are time-invariant. According to Stiglitz (2015), DSGE models can work well in normal times, but cannot handle crises. On the same line, Krugman (2011) points out that, not only orthodox macroeconomists did not forecast the crisis, but they did not even admit the possibility of such event. Even worse, they did not provide any useful advices to policy makers to put back the economy on a steady growth path.

As mentioned, DSGE models assume representative agents with rational expectations, which means that they do not systematically make mistakes and that they know exactly how the economic system works. According to Colander (2006), these models focus on the study of how globally rational agents with perfect information and foresight operate. It was felt that this way of treating the problem would provide important insights into situations where there was no perfect information. In other words, it means that these models try to study the coordination of an economy in which globally rational agents are optimizing in information-rich environments. The tricky aspect of this is that it is necessary to make various simplifying assumptions to make the problem tractable, which includes eliminating some aspects of the problem that could lead to multiple solutions. To achieve a unique solution, or at least a small number of solutions, the modeler must significantly limit the allowable interactions of heterogeneous agents. Generally, it has been done by focusing on models that include a single representative agent.

Such hypotheses are polemic to say the least. As Howitt (2012) states, individual rationality is not enough to make the system converge to the point of equilibrium. Moreover, as Caballero (2010) argues, it does not seem reasonable to assume that agents have all the informational set necessary to achieve equilibrium, particularly in periods of enormous structural change. Hendry and Minzon (2010) show that when structural breaks occur and have impacts on the economic trajectory, non-stationary elements may arise that prevent the economy from converging to equilibrium. Along the same line of thinking, Knight (1921) and Keynes (1936) stated that, in the presence of uncertainty, agents follow heuristic and non-optimizing rules, differently from what the authors of the neo-classical paradigm assume.

Given the need to respond to the above criticisms, the authors of the DSGE models sought to incorporate heterogeneous agents (Eggertsson and Krugman, 2012; Kumhof et al., 2015), bounded rationality (Branch and McGough, 2011; DeGrauwe, 2012; Anufriev, 2007), financial sector (Christiano et al., 2011 and 2013), and to investigate the consequences of rare events in the DSGE models, such as in Curdia et al. (2014), Fernandez-Villaverde and Levintal (2016). As for heterogeneity, the models contemplate certain types of agents exogenously determined (i. e., rich and poor) without interaction between them. When bounded rationality is incorporated, agents, for example, may be rational or not, and the dynamics of the economy is affected by increases in the percentage of agents that adopt different expectations rules. Regarding the inclusion of the financial sector, such models do not deeply analyze the issue of the endogenous money or the role of the interaction networks among banks. Finally, economic cycles still continue to be triggered by exogenous factors (Fagiolo and Roventini, 2012, 2017).

An increasing number of leading economists have claimed that the 2008 "economic crisis is a crisis for economic theory" (Kirman, 2010, 2016; Colander, 2006; Krugman, 2011; Caballero, 2010; Stiglitz, 2011, 2015; LeBaron and Tesfatsion, 2008). Their view is that the basic assumptions of DSGE models (e.g. rational expectations, representative agents, perfect markets etc.) prevent the understanding of basic issues underlying the current economic crisis and, more generally, macroeconomic dynamics.

This work embraces the idea that the economic system has its own properties, which emerge from the constituent social actions and interactions. It is characterized by being complex, because it is formed by heterogeneous elements that interconnect, with high multiplicity. The conception of economics as a system implies that economic functioning is not transparent to agents, that is, the latter are partially blind to macro events. In this sense, the economic system presents emergent properties, that is, characteristics that arise from the interaction between individuals that are not characteristic of the individual agents.

Under this approach, Agent-Based Models (ABMs), also called Agent-Based Computational Economics (ACEs), may represent an important research tool for the economic system. ABMs are the computational study of dynamic systems composed of interacting agents that can be modeled with bounded rationality (but not only) and not according to the optimizing behavior of the standard economy. The term "agent" can represent consumers, workers, households, companies, institutions, among others (Tesfatsion and Judd, 2006).

An ABM may include agents with learning ability that develops over time. In this way, ABMs seem able to respond in a more realistic way which aggregate behavior prevails when agents with not-so-rational behaviors are assembled as the standard economy assumes. Nonetheless, ABMs may be able to reproduce a wide array of micro and macro empirical regularities, including stylized facts concerning financial dynamics and banking crises, for instance.

The experiments of the ABMs seek to be based on more realistic assumptions regarding the behavior and interaction of the agents. In this sense, they seek to incorporate evidences from behavioral psychology (Kahneman and Tversky, 2000), and the super rationality of agents is replaced by bounded rationality and rational expectations by evolutionary-adaptive expectations. Analogously, network theory (e.g. Barabasi and Albert, 1999) may be included to investigate the micro and macroeconomic impact of interrelationships among various agents of the economic system.

As in Dawid et al. (2013), Fagiolo and Roventini (2012, 2017) and others, we believe that ABMs can provide new kinds of insights, thereby showing the potential of agent-based modeling as an instrument complementing established modeling approaches. In this sense, we defend that new developments and extensions of DSGE models are certainly welcome, but we insist that they should consider economy as a complex evolving system (i.e., as an ecology populated by heterogeneous agents whose far-from-equilibrium interactions continuously change the structure of the system). This is indeed the methodological core of ACE (Tesfatsion and Judd, 2006; LeBaron & Tesfatsion 2008).

However, ABMs are still in their infancy and some issues are still pending. The first one refers to the role played by micro and macro parameters. This is known as the problem of over-parametrization. It often appears because one typically inputs in the specification of agents' behavioral rules and interaction patterns many ingredients in order to meet as much as possible the reality. But it can be a problem. The interpretation of different parametrizations can be very difficult. Another issue relies on which set of parameters should be used to respond to policy implications. It is important to notice that these issues are closely related to a regular critique that ABMs usually face, as pointed out by Fagiolo and Roventini (2017, pp. 23-24), which is: “if an ABM contains many free parameters and it is able to reproduce a given set of stylized facts, how can one be sure that it represents the minimal mechanisms capable of reproducing the same set of stylized facts?”. Despite such a problem of over-parametrization is still not completely fixed, many developments have been made in the last years, as it will be shown in the third chapter.

The second issue concerns the role played by initial conditions. If these really matter, then one is required to identify the "true" set of initial conditions in the empirical data in order to correctly define such conditions for his/her model. There is a possibility of infinite regress. If this is the case, then the researcher may need data stretching back a very long time, possibly before data started to be collected! But how far should the researcher go in his quest to find such initial conditions empirically? There are also progresses regarding this issue.[2]

The second is closely related to the third issue, which regards the relation between simulated and real-world data. If, in principle, one could create as many theoretical observations as he likes, in practice one may only have a few (possibly only one!) of such empirical realizations.

It is worth noticing that the three issues explicated above affect any stochastic, dynamic (economic) model, DSGE-based ones included. Indeed, they are subject of many debates among philosophers of science (see Fagiolo et al., 2007).

The last issue is specific and concerns the comparability of different ABMs. Comparing DSGE models is easier because they are built using a well-stablished set of behavioral rules and their empirical validation is measured using common techniques (e. g. VAR models). In other words, we could say that there is a common guideline about how to do macroeconomics with DSGE models. On the other hand, “the lack of such a widespread agreement among the ACE community hinders the dialogue among different ABMs, reducing the comparability of their results, and possibly slowing down new developments” (Fagiolo and Roventini, 2017, p. 24). In this respect, there is still a lot to be done and it opens a great window of opportunity for future works (i.e., the development of common guidelines, dedicated languages and platforms, etc.).

[2] Comparing Fagiolo and Roventini (2017) to the situation discussed in Fagiolo and Roventini (2012), it is clear that some progress has been made in this respect, especially in the efforts devoted to identifying ergodicity tests for ABMs).

As we can see, Agent-Based simulation models are a relatively new addition to the tool-box of macroeconomics, but it has been shown that new kinds of insights can be obtained that complement established modeling approaches. The understanding that a more productive research avenue should avoid, when it is possible, the strong theoretical requirements of standard models (e.g., equilibrium, rationality, representative agent, etc.) means considering the economy as a complex evolving system. Once relying upon more realistic hypotheses - since it does not depend on equilibrium assumptions or fictitious auctioneers and does therefore not rule out coordination failures, instability and crisis by definition - an ABM can be an important step towards a better understanding of macroeconomics and its aggregates interrelations.

Moreover, many of ABMs are able to replicate the main stylized facts concerning business cycle fluctuations and to highlight the economic processes that generate these fluctuations, along with the attempt to match the micro-dynamics to reproduce the heterogeneity in firm investment behavior, firm distribution and income distributions.

However, complexity comes with a cost as ABMs fail to keep the analytical tractability of DSGE models and lack well-established econometric approach to estimation, as we will see. On the contrary, DSGE modelling has a consistent theoretical formalization that allows researchers to assess the effect of different economic hypotheses and parametric choices (Gobbi, A. and Grazzini, J., 2017). So, despite their empirical and theoretical limitations, the importance of DSGE models is huge - and mainly its extensions introduced after the Financial Crisis – for the modern macroeconomics.

Considering all the introductory discussion above, this work defends the idea that a more fruitful research avenue should consider a less insular modelling approach to macroeconomics. ABMs cannot afford to work in isolation. Neither can ABMs. The joint contribution between these two approaches can yield new developments and insights, especially when dealing with a world in recession. Moreover, we believe that exploiting diversity in macroeconomic modelling can benefit the profession and yield more fruitful developments regarding the formulation of macroeconomic policy.

One way of exploring diversity in macroeconomics is by exploring systematically both the DSGE and the ABMs, revealing their relative strengths and limitations, and combining these two different approaches, so that we can explore what one can learn from the other and perhaps yield a hybrid model. This is a very promising line of research. However, DSGE and Agent-Based literature streams have grown independently and it is very challenging to compare results and policy implications, as well as to combine them into a common model. Our ambitious here is lower.

The effort of this work will be to propose a systematic investigation of the two – still competing – modeling approaches. For this end, in the first chapter it will be shown the main assumptions and ideas behind the DSGE methodology, as well as an analytical investigation on how these models are built. The second chapter introduces the main criticisms raised against the DSGE framework and how ABMs can contribute to a more realistic modelling approach. By exploring the main features and building structures of ABMs, chapter two also deals with the issue of estimation and validation. The last section concludes by summarizing and comparing the main ideas of both methodologies and proposes promising future researches.

2. UNDERSTANDING DSGE

In this chapter, we intend to explain DSGE models in a very particular way, i. e., allowing readers to overcome some of the main difficulties regarding the comprehension of these types of models. Although DSGE modelling has become very popular in modern macroeconomics, it strikes us that in the current economic literature there is a lack of manual that reveals – step-by-step – how this line of research works.

Generally, papers begin with a presentation of the agents' objective functions and of the equations solutions of the maximization problem, whilst no resolution is shown. By doing that, it becomes difficult to properly identify the exact theoretical model and its application (Costa Jr., 2016).

In short, DSGE combines rational expectations of all agents with price stickiness (Carlin and Soskice, 2015), as they are derived from the combination between RBC and NK-models. Thus, understanding DSGE requires us to comprehend its roots, that is, both the RBC and the NK modelling framework.

Of course, this work does not encompass the whole DSGE methodology, but it presents in good details how these models are constructed, how they work, what are their underlying assumptions as well as their limitations and criticisms. Along with that, we try to point out some of the extensions and ramifications within this approach.

To do so, in what follows, we start with a short explanation of some of the main assumptions and principles adopted by DSGE models, and proceed with both a theoretical and a modelling demonstration of RBC and NK approaches. Finally, we hope that all this effort allows readers to understand DSGE and to compare its methodology with Agent-Based modelling.

2.1 The Idea of Representative Agents

Agents in an economy are heterogeneous, i. e., every consumer has different preferences for goods and services and every firm has different preferences regarding the use of technology in their production process. Nevertheless, considering these characteristics when modelling an economy creates a potential theoretical problem, how can one properly identify each economic agent's individual choices?

It is a fact that any economic (and models in general) is a simplified description of a certain complex phenomenon. However, it seems obvious that it would be impossible to identify each individual agent's exact choices. The way out of this issue by the standard economic approach was to group economic agents into larger categories[3]. By doing that, it is assumed that a large quantity of identical agents exists in the economy.

This procedure within DSGE approach is called introducing a representative agent. It is indeed a significant simplification of a complex reality; however, defenders of this methodology advocate that macroeconomic modelling is a lot simpler by adopting such procedure, as it fulfills the purpose of macroeconomic studies, such as how household's consumption responds to a rise in the interest rates. Moreover, they argue that the aim of DSGE modelling is to build relatively small theoretical problems. Thus, when aggregating the representative agents' behavior, one can properly study how they interact, allowing for a detailed analysis of macroeconomic policy effects.

2.2 The Idea of Agents' Lifespan

Agent's lifespan means the temporal reference that agents use to make their decisions. For the purposes of DSGE models, it is assumed that agents have infinite time horizons. Again, this is a very simplified description of the reality, once firms, consumers and governments do not exist forever.

The explanation for this assumption is quite simple. For example, when a government decides upon its budget, it does not expect a government collapse or any kind of event that can make it cease to exist. Firms act likewise, as they decide their budgets not considering that they will go bankrupt in the future. Regarding consumers, although it is assumed that each one has a finite lifespan, when considering the whole family structure, the "family representative agent" becomes infinite, as their members will periodically born and die.

[3] For example, in a survey regarding consumers' behavior, the modeller could create groups with similar consumption characteristics (e.g. low-, average-, and high income consumers).

2.3 Real Business Cycle (RBC)

2.3.1 Introduction

The RBC model was developed from the New Classical Macroeconomics[4]. The authors of the RBC methodology wanted to craft a model of business cycles based on the neo-classical growth model, so that they could be able to put together both a model of cycles and growth with intra- and inter-temporally optimizing agents.

To do so, they took the Ramsey version of the Solow model[5] and added shocks to technology and rational expectations. The agents' behavior is incorporated by the so-called "deep parameters", which characterize both the production and utility functions (Carlin and Soskice, 2015). An important feature of these macro models is that they are policy-invariant:

> A fundamental feature of this approach is that business cycles arise because of exogenous technology shocks, and that these shocks result in economic fluctuations because of the way agents respond to the new opportunities they face as a consequence of the shocks. For example, following a negative technology shock, which reduces real wages, the economy is in a business cycle trough and the reduction in aggregate hours worked in the economy is the outcome of employees choosing to supply less labor. Since fluctuations in employment are due to choices made by workers about their labor supply, the unemployment in a business cycle trough is voluntary. When cycles are equilibrium phenomena, as they are in the RBC framework, there is no presumption that policy intervention to 'stabilize' the economy would improve welfare (Carlin and Soskice, 2015, p. 585).

The name "RBC" comes from the source of the fluctuations, which is on the supply side - i. e., it is deviations in the production function (technological shocks) what causes fluctuations in economic activity that are observed in the real world. Furthermore, what turns random shocks to technology into business cycle fluctuations in the RBC models is the way that households respond to changes in the real wage and the real interest rate. This is the major propagation mechanism in these models.

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[4] The early developers of this approach were Robert Lucas (1972), Thomas Sargent and Neil Wallace (1975), Finn Kyland and Edward Prescott (1977).

[5] The Ramsey version is when a constant savings ratio is replaced by optimizing households that choose their saving rate, i. e., when saving rate is endogenous (See Carlin and Soskice, 2015, chapter 4, for a detailed explanation).

Therefore, household consumption and labor supply are at the heart of this framework. In what follows, we shall further understand how the above features work in practice by modelling a basic structure compounded of two types of agents and two types of goods.

2.3.2 Basic Structure of RBC Models

For the purposes of this chapter, it will be considered a ‘basic’ model compounded of two types of agents: households and firms[6]. It will be assumed *perfect competition* and *fully flexible prices* in all markets.

In section 2.1 we saw that, in the real world, although there is a very large number of households, they can be treated as if they were somehow identical. The same logic applies for the firms – i.e., they are treated as if they had the same technology. Again, this procedure is called introducing a representative agent, in this case, a representative firm.

In order to show the basic ideas behind these types of models, this section will demonstrate: a) how households solve two problems of choice: 1) the intratemporal choice between consumption and leisure and 2) the intertemporal choice between consumption and saving; and b) how firms choose the inputs that will be used in their production process. As we shall note, basically in all cases, the marginal rate of substitution is compared to relative price.

2.3.2.1 A model of two goods: consumption and leisure

Our interest is to study how consumers choose what they consume in an economy where there are two large categories of consumer goods: ‘good 1’ and ‘good 2’. However, before doing this, we must define how these consumers earn their income.

Let us assume that consumers obtain all their income from their labor. Thus, one can choose to work a certain number of hours – receiving a wage W per hour – and to rest what remains of his/her available time. Presumably, work is a consumer ‘bad’ – i.e., agents do not wish to work a lot, because the more they do, the less time they have for leisure.

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[6] It is worth saying that a ‘complete’ model would consist of five agents: 1) households, 2) firms, 3) fiscal and monetary authorities, 4) the foreign sector and 5) financial institutions.

Adapting these characteristics to the standard consumer theory requires us, instead of considering work a ‘bad’, to consider leisure a ‘good’. So, we define leisure as the number of hours left after subtracting the time spent working from the total number of available hours in a certain period:

$$Leisure + Work = h\ (avaiable\ time) \tag{1}$$

2.3.2.1.1 Indifference Curves (consumption-leisure)

In our way to understand consumers’ choices, we also need to understand if they would be willing to give up leisure for consumption (or vice versa) and at what rate. For that, we need a definition of an indifference curve.

Consider a certain consumption bundle (x_1, x_2). Now, if we shade in all the consumption bundles that are weakly preferred to (x_1, x_2), we will have a weakly preferred set. All the bundles on the boundary of this set form the indifference curve. In other words, the indifference curve shows the bundles for which the consumer is just indifferent to (x_1, x_2). (For further details, see Varian, 2010, pp. 36-48).

Note that, here, we are considering that the two factors that give utility to an individual are consumption (C) and leisure $(H) - u(C, H)$, both treated as goods. The utility function is assumed to have two properties:

1) Strictly increasing $\left(\frac{\partial u}{\partial C} > 0\ ;\ \frac{\partial u}{\partial H} > 0\right)$
2) Diminishing marginal returns $\left(\frac{\partial^2 u}{\partial C^2} < 0\ ;\ \frac{\partial^2 u}{\partial H^2} < 0\right)$

With these assumptions and the definition of an indifference curve, it is possible to plot an indifference curve map. The indifference curve map follows the standard properties of consumer theory, which is: each curve has a negative slope and is convex to the origin. Furthermore, the curves may not cross each other.

2.3.2.1.2 Marginal Rate of Substitution (MRS)

With the above definitions and assumptions, we can measure how many units of a good an individual is willing to give up in exchange for another good. In our model, it means measuring how many units of leisure a consumer would accept to give away in exchange for more units of consumption.

On a graph formed by two goods (e.g. X and Y), the MRS is the negative slope of an indifference curve at some point. That is:

$$MRS_{X,Y} = -\frac{\partial Y}{\partial X}\bigg|_{U=U_1}$$

$$MRS_{X,Y} = -\frac{MU_X}{MU_Y}\bigg|_{U=Ui}$$

Where MU_X and MU_Y represent the marginal utilities in relation to goods X and Y, respectively, and $|U = U_i$ notation indicates that the slope is calculated along the indifference curve U_i.

2.3.2.1.3 Budget Constraints

Studying a consumer's optimal choice requires more than the indifference map. It is deductible that each individual has a total amount of income he/she can spend, depending on how much he/she chooses to work.

Let us suppose that an individual has 60 hours per week available for work and leisure. Moreover, assume that this individual can work how many hours he/she wants, receiving an hourly wage W. So, the total weekly income would be:

$$Y = L.W \tag{2}$$

From (1), we have therefore $\boldsymbol{h = 60}$, i.e., 60 hours available per week. So, $L = 60 - H$. Thus, income can be written as a function of leisure:

$$Y = (60 - H).W \tag{3}$$

Here, another assumption is taken. We will consider that consumers spend all the income they receive, not saving anything. Each consumer good (c) is available on the market for the price (P). Therefore, an individual's consumption in each period is:

$$P.c = Y \tag{4}$$

Combining (3) and (4), we arrive at the following budget constraint:

$$P.c = (60 - H).W \tag{5}$$

In expression (5), an individual takes prices of consumer goods (P) and hourly wages (W) as given. He/She can only choose the level of consumption and the amount of leisure.

Rearranging (5), we have:

$$P.c = (60.W) - (H.W) \rightarrow \overbrace{\underbrace{P.c}_{\substack{Consumer\\Goods}} + \underbrace{W.H}_{Leisure}}^{\substack{Destination\ of\\Income}} = \underbrace{60.W}_{\substack{Total\ Disposable\\Income}} \tag{6}$$

From (6), we can see that the total disposable income $(60.W)$ is used to acquire consumer goods $(P.c)$ and leisure $(W.H)$. Although leisure cannot be directly bought or sold on the market, we can understand wages as the leisure cost of opportunity. Each hour spent on leisure is an hour that could have been spent working. Thus, if we explicitly consider the opportunity costs, wages are thus the price of leisure.

2.3.2.2 Obtaining Individuals' Optimal Choices

Our effort to explain individual's preferences and budget constraints was intentional. It is the interaction between them that will give us an individual's optimal choice. Formally speaking, an individual faces the following optimization problem:

$$\max_{c,L} u(c, L)$$

Subject to,

$$P.c = W.L \tag{7}$$

We will use the Lagrangian method to solve this type of problem. The Lagrangian is:

$$\mathcal{L} = u(c, L) - \lambda(P.c - W.L) \tag{8}$$

The first-order conditions for c and L are:

$$\frac{\partial \mathcal{L}}{\partial c} = \frac{\partial u}{\partial c} - \lambda P = 0 \tag{9}$$

$$\frac{\partial \mathcal{L}}{\partial L} = \frac{\partial u}{\partial L} + \lambda W = 0 \tag{10}$$

Combining (9) and (10), we have:

$$\frac{\frac{\partial \mu}{\partial L}}{\frac{\partial \mu}{\partial c}} = -\frac{W}{P} \tag{11}$$

Note that the left-hand side of the last expression represents the leisure-consumption's marginal rate of substitution, whilst the right-hand side is the leisure-consumption relative price.

In summary, the problem of the household is to maximize utility given a fixed amount of income. For this accomplishment, an individual will buy the amount of goods that depletes his/her total income equating to the physical rate of tradeoff between any two goods (MRS) and the rate at which a good can be exchanged for another on the market-relative price. The optimal consumption bundle is the point that represents the pair of goods that is on the highest indifference curve and is within the individual's budget constraint.

2.3.2.3 Dynamic Structure of Consumption-Savings

Whenever an individual makes a choice between consumption and leisure in the current period, he/she often recognizes that a similar choice will be made sometime later. To formalize this, economists use a utility function and almost always simplify intertemporal problems assuming that preferences are additively separable:

$$u\left(c_1, c_2, c_{3,} \ldots\right) = u(c_1) + \beta u(c_2) + \beta^2 u(c_3) + \cdots \tag{12}$$

with $\beta > 1$.

The β parameter is called an intertemporal discount factor. It is less than one as it represents that households are more concerned with present than future consumption[7].

For the sake of clarity, we will ignore data regarding leisure, as it follows the same logic, and assume that individuals live only in two periods: the present (period 1) and the future (period 2). As Costa Jr. (2016, p. 22) argues: “this division into two periods is enough to illustrate the basic principles of macroeconomic events that occur intertemporally in a structure with an infinite time horizon”.

In what follows, we assume all the usual properties of the utility function. In this simplified RBC model, individuals receive income twice during their lives – once in period 1 and once in period 2. They start off in period 1 with a certain amount of wealth (A_0) and choose how much they want to consume, paying a price of P_1, as well as how much wealth they will carry forward to period 2 (A_1). A_0 and A_1 may assume negative values, which indicate that an individual would be a borrower in these periods.

The budget constraint in period 1 can be written as:

$$P_1 . c_1 + A_1 = R. A_0 + Y_1 \ , \tag{13}$$

where R is the gross nominal interest rate[8] that represents the returns on each monetary unit held as a financial asset from a period to another.

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[7] $\beta = \frac{1}{1+\theta}$,where θ is the subjective intertemporal preference rate. This parameter indicates the value of future utility in relation to present utility. The greater the value of β, the more patient the household is with regard to consumption (Costa Jr., 2016, p. 22).

[8] A gross rate is defined as $R = 1 + r$, where r is the net return for the period.

An individual's budget constraint in period 2 follows the same logic, but the final wealth must be zero, as individuals live only for two periods.

Before continuing, let us define periods' savings as the difference between total income and the total spent within a given period, that is:

$$S_1 = (R - 1).A_0 + Y_1 - P_1.c_1 \tag{14}$$

Rearranging (14):

$$A_1 - A_0 = (R - 1).A_0 + Y_1 - P_1.c_1 \tag{15}$$

Note that: $S_1 = A_1 - A_0$, i. e., the amount of individuals' savings in period 1 is equal to the variation in their wealth within the period. Similarly: $S_2 = A_2 - A_1$.[9]

By combining the budget constraints of period 1 and 2, we arrive at an individual's intertemporal budget constraint.

Solving the budget constraint for A_1 on period 1, we have:

$$A_1 = R.A_0 + Y_1 - P_1.c_1 \tag{16}$$

Substituting (16) in period 2's budget constraint:

$$P_2.c_2 = R.[R.A_0 + Y_1 - P_1.c_1] + Y_2 \tag{17}$$

Dividing both sides by R:

$$P_1.c_1 + \frac{P_2.c_2}{R} = Y_1 + \frac{Y_2}{R} + R.A_0 \tag{18}$$

The right-hand side of (18), which considers the initial wealth and the individual's lifetime income, represents the discounted intertemporal resource. The left-hand side

[9] It is worth saying that it is an approximation to general economic behavior to assume that individuals are rational during their lifespans, saving and/or borrowing appropriately during their entire lives.

represents the discounted intertemporal consumption, which considers the consumption in all periods.

Assuming that initial wealth is zero ($A_0 = 0$) and solving (18) for c_2, we have:

$$c_2 = \left[\left(\frac{R}{P_2}\right).Y_1 + \frac{Y_2}{P_2}\right] - \left[\frac{P_1.R}{P_2}\right].c_1 \tag{19}$$

2.3.2.4 Optimal Intertemporal Choice [10]

In this consumption-leisure basic model, we are assuming that an individual lives only for two periods. Thus, preferences can be reduced to:

$$u(c_1, c_2) = u(c_1) + \beta.u(c_2) \tag{20}$$

Note that we can assume $A_2 = 0$, since keeping assets in the form of savings in period 2 would not be logical/optimal. Thus, an individual's budget constraints in both periods are:

$$P_1.c_1 + A_1 = R.A_o + Y_1 \tag{21}$$

$$P_2.c_2 = R.A_1 + Y_2 \tag{22}$$

Any individual faces the following problem:

$$\max_{c_1.c_2,A_1} \mu(c_1) + \beta.\mu(c_2)$$

subject to:

$$P_1.c_1 + A_1 = R.A_0 + Y_1$$

$$P_2.c_2 = R.A_1 + Y_2 \tag{23}$$

[10] An individual's optimal intertemporal choice is an interaction between his/her indifference curve map and intertemporal budget constraints (Costa Jr., 2016).

That is, the individual has to choose the levels of consumption for both periods and the level of wealth that maximizes his/her utility function, which is restricted by the budget constraints in both periods. The values of P and R are given.

The Lagrangian for this problem is:

$$\mathcal{L} = u(c_1) + \beta.u(c_2) - \lambda_1[P_{1.}c_1 + A_1 - R.A_0 - Y_1] \\ - \lambda_2[P_2.c_2 - R.A_1 - Y_2] \tag{24}$$

The first-order conditions for c_1, c_2 and A_1 are:

$$\frac{\partial\mathcal{L}}{\partial c_1} = \frac{\partial u}{\partial c_1} - \lambda_1.P_1 = 0 \tag{25}$$

$$\frac{\partial\mathcal{L}}{\partial c_2} = \beta\frac{\partial u}{\partial c_2} - \lambda_2.P_2 = 0 \tag{26}$$

$$\frac{\partial\mathcal{L}}{\partial A_1} = -\lambda_1 + \lambda_2.R = 0 \tag{27}$$

Rewriting (25), we have: $\lambda_1 = \frac{\partial u/\partial c_1}{P_1}$

Rewriting (26), we have: $\lambda_2 = \beta\frac{\partial u/\partial c_2}{P_2}$

Now, substituting these results in (27) and defining $\pi_2 = \frac{P_2}{P_1}$, we arrive at the following expression:

$$-\frac{\partial\mu/\partial c_1}{\beta\partial\mu/\partial c_2} = -\frac{R}{\pi_2} \tag{28}$$

Equation (28) is called the Euler Equation. Note that it relates the marginal utility of consumption for both periods with the relative price of intertemporal consumption[11].

[11] It is worth remembering that the indifference curve's slope measures the extra consumption that would be necessary in the following period to offset the loss of a unit of consumption in the current period. Contrariwise, the budget constraint's slope determines the premium R for saving more. The slopes of the indifference curves and the budget constraint are equal. Besides, higher the values for β (i.e., patient individuals), lower the slopes of the indifference curves (Barro, 1997; Costa Jr., 2016). For a graphical example, see Costa Jr (2016, pp. 28-29).

Following the standard approach, the next step is to model how firms choose inputs and production levels with the objective of maximizing economic profit. Briefly speaking, firms wish to obtain the largest possible difference between revenue and total costs. For the sake of briefness, we will omit this formalization[12].

For our purposes, what we need to know is that firms will choose input levels so that the marginal product of these inputs equals their real marginal costs. As Costa Jr. (2016, p. 32) states:

> Agents, when deciding upon their choices, use marginal rates of substitution between goods and their relative prices. First, households must face the consumption-leisure tradeoff analyzing the relative price between these goods (real wages). When the choice is intertemporal, the tradeoff is between consumption today and future consumption, and the relative price is the nominal interest rate. Firms must make the same type of decision when deciding the combination of units of labor and capital to be used, analyzing the relative prices of these inputs $\left(\frac{W}{R}\right)$.

2.3.3 The Model[13]

Let us now present the structural formalization of the RBC model proposed. To do so, we will start with the presentation of the agents, showing the equilibrium conditions. Then, we will find the steady state and log-linearize the model's equilibrium equations.

The RBC model proposed requires some assumptions. First, the economy is closed, with no government or financial sector. Second, the economy does not have a currency. Third, adjustment costs do not exist. As we will see, in the basic New Keynesian model (NK-model) presented in section 2.4, the first two assumptions will continue valid.

2.3.3.1 Households

We assume that the economy is composed by a unique set of households indexed by $j \in [0,1]$. Their problem is to maximize a particular intertemporal welfare function,

[12] The formalization is well demonstrated in Costa Jr. (2016, pp. 29-32).

[13] We are considering the model presented in Costa Jr (2016, chapter two). However, it is a very simplistic model. It is worth noting that RBC models have become much more sophisticated in recent years. For a different modelling approach, see the baseline RBC model proposed in Romer (2012, pp. 195-233).

that is, a utility function additively separable into consumption (C) and labor (L). Thus, further assumptions must be considered.

First, consumption is intertemporally additively separable, in the sense that there is no habit formation[14]. Second, population growth is not considered. With these assumptions considered, the representative household faces the following problem:

$$\max_{C_{j,t1}.L_{j,t},K_{j,t+1}} E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \frac{L_{j,t}^{1+\varphi}}{1+\varphi} \right) \tag{29}$$

Subject to:

$$P_t\left(C_{j,t} + I_{j,t}\right) = W_t.L_{j,t} + R_t.K_{j,t} + \pi_t \tag{30}$$

In (29), E_t is the expectations operator, β is the intertemporal discount factor, C is the consumption of goods, L is the number of hours worked, σ is the relative risk aversion coefficient[15], and φ is the marginal disutility of labor supply.

The utility function must have the same characteristics we mentioned before, i. e.: $U_C > 0;\ U_L < 0; U_{CC} < 0; U_{LL} < 0$ [16]. It means that consumption has a positive effect on the utility of household, but labor does not. Also, the utility function is concave, which represents the fact that, as consumption increases, so does utility, but at lowers rates.

In (30), P is the general price level, I is the level of investment, W is the level of wages, K is the capital stock, R is the return on capital, and π is the firms' profits (dividends).

Households maximize their welfare function, which is subject to their intertemporal budget constraints. It is assumed that households are the owners of all the

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[14] Empirical evidences show that households do not change their pattern of consumption right after an unexpected shock alters their income, because they use their savings to mitigate their loss. In the economic literature, this friction is known as habit formation or consumption habits. As we will see in section 2.5, some NK-DSGE models address this empirical evidence that the behavior of the households follows a regular pattern. So, they consider an intertemporally non-separable utility function.

[15] Note that we are assuming a constant relative risk aversion (CRRA) utility function, since it is the most common when representing household choices. However, there are other functions used in the literature. See Hansen (1985) and Gertler and Karadi (2011) for examples of a logarithmic utility function and a combination of log and CRRA, respectively.

[16] U_C and $U_L$are the first-order derivatives of the utility function in relation to consumption and labor, respectively. U_{CC} and U_{LL} are the second order derivatives.

economy's factor of production – capital and labor. When providing labor and capital to firms, households receive wages and return on capital. They also own the firms, so they receive dividends.

Finally, the following expression shows capital accumulation over time:

$$K_{j,t+1} = (1-\delta)K_{j,t} + I_{j,t} \ , \tag{31}$$

where δ is the depreciation rate of physical capital.

The Lagrangian for this maximization problem is:

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} \beta^t \left\{ \left[\frac{C_{j,t}^{1-\sigma}}{1-\sigma} - \frac{L_{j,t}^{1+\varphi}}{1+\varphi} \right] - \lambda_{j,t} [P_t . C_{j,t} + P_t . K_{j,t+1} - P_t . (1-\delta) K_{j,t} - W_t . L_{j,t} - R_t . K_{j,t} - \pi_t] \right\} \tag{32}$$

The first-order conditions for C, L and K are:

$$\frac{\partial \mathcal{L}}{\partial C_{j,t}} = C_{j,t}^{-\sigma} - \lambda_{j,t} . P_t = 0 \tag{33}$$

$$\frac{\partial \mathcal{L}}{\partial L_{j,t}} = -L_{j,t}^{-\varphi} + \lambda_{j,t} . W_t = 0 \tag{34}$$

$$\frac{\partial \mathcal{L}}{\partial K_{j,t+1}} = -\lambda_{j,t} . P_t + \beta . E_t . \lambda_{j,t+1} [(1-\delta) E_t . P_{t+1} + E_t . R_{t+1}] = 0 \tag{35}$$

Solving for λ_t, we arrive at the household's labor supply equation:

$$C_{j,t}^{\sigma} . L_{j,t}^{\varphi} = \frac{W_t}{P_t} \tag{36}$$

Multiplying both sides by -1:

$$\overbrace{-C_{j,t}^{\sigma}.L_{j,t}^{\varphi}}^{\substack{consumption-leisure\\MRS}} = \overbrace{-\frac{W_t}{P_t}}^{\substack{consumption-leisure\\relative\ price}} \tag{37}$$

Expression (37) shows that the consumption-leisure relative price (i.e. real wage) is equal to the leisure-consumption marginal rate of substitution[17]. It means that a rise in consumption will be only possible with a rise in the number of labor hours (less leisure), *ceteris paribus*. On the other hand, with higher wages, it is possible to increase consumption without giving up leisure.

From (33), we arrive at:

$$\lambda_{j,t} = \frac{C_{j,t}^{-\sigma}}{P_t}$$

So,

$$\lambda_{j,t+1} = \frac{C_{j,t+1}^{-\sigma}}{P_{t+1}}$$

Substituting these results in equation (35), we have:

$$-C_{j,t}^{-\sigma} + \beta.E_t\left\{\left(\frac{C_{j,t+1}^{-\sigma}}{P_{t+1}}\right)[(1-\delta)P_{t+1} + R_{t+1]}\right\} = 0$$

$$\left(\frac{E_t.C_{j,t+1}}{C_{j,t}}\right)^{\sigma} = \beta\left[(1-\delta) + E_t.\left(\frac{R_{t+1}}{P_{t+1}}\right)\right] \tag{38}$$

Equation (38) is the Euler equation, which determines the household's savings decision[18]. It means that when deciding their level of savings, households compare the utility rendered when consuming an additional amount today with the utility that could be rendered by consuming more in the future. So, if interest rate expectations rise, present consumption (at t) will be more expensive, so future consumption (at $t+1$) will rise, *ceteris paribus*.

[17] Note that it follows the same logic as in expression (11).
[18] In this model, savings is the acquisition of investment goals.

The household's problem, in Costa Jr's words:

> (…) boils down to two choices. The first is an intratemporal choice between acquiring consumption and leisure goods. The other is an intertemporal choice, in which the household must choose between present and future consumption" (Costa Jr., 2016, p. 37).

2.3.3.2 Firms

Representative firms are agents that produce goods and services that can be consumed or saved by households[19]. The first assumption we need to consider is that there is a continuum of firms indexed by j that maximize profits in a perfect competition market structure. This means that their profits will always be zero($\pi_t = 0;\ \forall t$).

For our purposes, we are assuming a Cobb-Douglas production function:

$$Y_{j,t} = A_t . K_{j,t}^{\alpha} . L_{j,t}^{1-\alpha} \tag{39}$$

A_t represents productivity; Y_t represents the product and α is the elasticity of the level of production with respect to capital. α can also be thought of as the participation of capital in income. Thus, $(1-\alpha)$ would be the labor level of participation.

The firm faces the problem of maximizing the profit function, choosing the ideal amounts of both inputs (capital and labor). That is:

$$\max_{L_{j,t}, K_{j,t}} \pi_t = A_t . K_{j,t}^{\alpha} . L_{j,t}^{1-\alpha} . P_{j,t} - W_t . L_{j,t} - R_t . K_{j,t} \tag{40}$$

The first-order conditions for $K$and L are:

$$\frac{\partial \pi_{j,t}}{\partial K_{j,t}} = \alpha A_t . K_{j,t}^{\alpha-1} . L_{j,t}^{1-\alpha} . P_{j,t} - R_t = 0 \tag{41}$$

$$\frac{\partial \pi_{j,t}}{\partial L_{j,t}} = (1-\alpha) A_t . K_{j,t}^{\alpha} . L_{j,t}^{-\alpha} . P_{j,t} - W_t = 0 \tag{42}$$

[19] When households choose to save, they will eventually transform goods/services into capital.

Rearranging (41), we arrive at:

$$\frac{R_t}{P_{j,t}} = \alpha . \frac{Y_{j,t}}{K_{j,t}} \tag{43}$$

Expression (43) represents the demand for capital. Note that $\left(\frac{R_t}{P_{j,t}}\right)$ is the real marginal cost of capital and $\left(\alpha . \frac{Y_{j,t}}{K_{j,t}}\right)$ is the marginal product of capital.

Now, rearranging (42):

$$\frac{W_t}{P_{j,t}} = (1-\alpha) . \frac{Y_{j,t}}{L_{j,t}} \tag{44}$$

Expression (44) represents the demand for labor. Again, note that $\left(\frac{W_t}{P_{j,t}}\right)$ is the real marginal cost of labor and $(1-\alpha) . \frac{Y_{j,t}}{L_{j,t}}$ is the marginal product of labor.

We will assume that productivity shocks follow a first-order autoregressive process:

$$\log A_t = (1-\rho_A) . \log A_{SS} + \rho_A . \log A_{t-1} + \epsilon_t \ , \tag{45}$$

where ρ_A is the autoregressive parameter of productivity, with $-1 < \rho_A < 1$ to ensure the stationary state of the process; A_{SS} is the value of productivity at the steady state and $\epsilon_t \sim N(0, \sigma_A)$, that is, ϵ_t is a white-noise disturbance – a series of mean-zero shocks that are uncorrelated with one another.

Following the RBC approach, prices levels must be equal to marginal costs. To obtain marginal cost, we must first combine equations (43) and (44):

$$-\frac{W_t}{R_t} = \frac{(1-\alpha) K_{j,t}}{\alpha L_{j,t}} \tag{46}$$

The first term of expression (46) represents the economic rate of substitution, which measures – maintaining the same cost – the rate at which labor can be replaced by capital. The second term represents the marginal rate of technical substitution, which measures – while maintaining a constant level of production – the rate at which labor can be replaced by capital.

Rearranging (46):

$$L_{j,t} = \left(\frac{1-\alpha}{\alpha}\right).\frac{R_t}{W_t}.K_{j,t} \tag{47}$$

Substituting (47) in the production function (39), we get:

$$Y_{j,t} = A_t.K_{j,t}^{\alpha}\left[\left(\frac{1-\alpha}{\alpha}\right).\frac{R_t}{W_t}.K_{j,t}\right]^{1-\alpha} \tag{48}$$

Rearranging (48):

$$K_{j,t} = \frac{Y_{j,t}}{A_t}\left[\left(\frac{\alpha}{1-\alpha}\right).\frac{W_t}{R_t}\right]^{1-\alpha} \tag{49}$$

Now, substituting (49) in (47):

$$L_{j,t} = \frac{Y_{j,t}}{A_t}\left(\frac{1-\alpha}{\alpha}\right)\frac{R_t}{W_t}\left[\left(\frac{\alpha}{1-\alpha}\right).\frac{W_t}{R_t}\right]^{1-\alpha}$$

$$\left(\frac{1-\alpha}{\alpha}\right)\frac{R_t}{W_t} = \left[\left(\frac{\alpha}{1-\alpha}\right).\frac{W_t}{R_t}\right]^{-1}$$

$$L_{j,t} = \frac{A_t}{Y_{j,t}}\left[\left(\frac{\alpha}{1-\alpha}\right).\frac{W_t}{R_t}\right]^{-\alpha} \tag{50}$$

Firms' total cost is represented by:

$$TC_{j,t} = W_t.L_{j,t} + R_t.K_{j,t} \tag{51}$$

Replacing (49) and (50) into (51):

$$TC_{j,t} = W_t.\frac{Y_{j,t}}{A_t}\left[\left(\frac{\alpha}{1-\alpha}\right).\frac{W_t}{R_t}\right]^{-\alpha} + R_t.\frac{Y_{j,t}}{A_t}\left[\left(\frac{\alpha}{1-\alpha}\right).\frac{W_t}{R_t}\right]^{1-\alpha} \tag{52}$$

With a little algebraic handling, we arrive at:

$$TC_{j,t} = \frac{Y_{j,t}}{A_t}.\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}.\left(\frac{R_t}{\alpha}\right)^{\alpha} \tag{53}$$

Knowing that the marginal cost (MC) is the first derivative of the total cost, we have:

$$MC_{j,t} = \frac{1}{A_t}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}.\left(\frac{R_t}{\alpha}\right)^{\alpha} \tag{54}$$

Note that the marginal cost depends only on the productivity and on the prices of the factors of production. So, it will be the same for every firm$(MC_{j,t} = MC_t)$.

Knowing that $P_t = MC_t$, we can arrive at the general price level of the economy:

$$P_t = \frac{1}{A_t}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t}{\alpha}\right)^{\alpha} \tag{55}$$

2.3.3.3 The Equilibrium Conditions

Now that we have described agents' behavior, we need to study the interaction between them in order to find macroeconomic equilibrium. In this model, households decide how much to consume (C), how much to invest (I) and how much labor to supply (L). Their goal is to maximize utility, taking prices as given. Firms decide how much to produce (Y) with the available technology. They can choose the levels of capital and labor, taking prices as given.

The equilibrium of the system consists of three blocks:

- A price system: W_t, R_t and P_t;
- An endowment of values for goods and inputs: $Y_t, C_t, I_t, L_t \; and \; K_t$;

- A production-possibility frontier that follows the equilibrium of the good market (i.e., aggregate supply = aggregate demand): $Y_t = C_t + I_t$

Finding a sequence of endogenous variables such that the equilibrium conditions are satisfied is what makes a competitive equilibrium for the RBC models.

2.3.3.4 Steady State

The next step after defining the equilibrium is to define the steady state values[20]. Some endogenous variables have their steady state values exogenously determined, as it occurs with productivity – the source of standard RBC models' shocks. At the steady state, $E(\varepsilon_t) = 0$. Note that, in equation (45), we do not know the value of A_{SS}, so the literature almost always assume $A_{SS} = 1$, and, therefore, $\log A_{SS} = 0$.

Removing variables' time indicators, the structural model is shown at Table 1.

[20] Definition: an endogenous variable is at the steady state in each t, if $E_t x_{t+1} = x_t = x_{t-1} = x_{SS}$ (Costa Jr., 2016).

Table 1 - Structure of the Model

Equation	Definition
$C_t^{\sigma}.L_t^{\varphi} = \frac{W_t}{P_t}$	Labor Supply
$\left(\frac{E_t.C_{j,t+1}}{C_{j,t}}\right)^{\sigma} = \beta\left[(1-\delta) + E_t.\left(\frac{R_{t+1}}{P_{t+1}}\right)\right]$	Euler Equation
$K_{t+1} = (1-\delta)K_t + I_t$	Capital's Law of motion
$Y_t = A_t.K_t^{\alpha}.L_t^{1-\alpha}$	Production Function
$K_t = \alpha.\frac{Y_t}{\frac{R_t}{P_t}}$	Demand for Capital
$L_t = (1-\alpha).\frac{Y_t}{\frac{W_t}{P_t}}$	Demand for Labor
$P_t = \frac{1}{A_t}\left(\frac{W_t}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t}{\alpha}\right)^{\alpha}$	Price Level
$Y_t = C_t + I_t$	Equilibrium Condition
$\log A_t = (1-\rho_A).\log A_{SS} + \rho_A.\log A_{t-1} + \epsilon_t$	Productivity Shock

Source: Adapted from Costa Jr. (2016, p. 42)

The system of equations in Table 2 will be used to determine the following endogenous variables at the steady state: $Y_{SS}, C_{SS}, I_{SS}, K_{SS,}L_{SS}, W_{SS}, R_{SS}$ and P_{SS}. Prices are the first values that must be found (W_{SS}, R_{SS} and P_{SS}).

To do so, we must consider the Walras' Law, which states that for any price vector $\boldsymbol{p}$ there is $\boldsymbol{pz}(\boldsymbol{p}) \equiv 0$ (i.e., the demand excess value is identically zero). In short, considering the Walras' Law, the economy's general price level can be normalized ($P_{SS} = 1$).[21]

[21] See Costa Jr. (2016, p. 43) for a proof of the Walras' Law and how the economy's general price can be normalized ($P_{SS} = 1$).

Table 2 - Model's equations at the steady state

Households		Firms		Equilibrium Condition	
$C_{SS}^{\sigma}.L_{SS}^{\varphi} = \frac{W_{SS}}{P_{SS}}$	(56)	$K_{SS} = \alpha.\frac{Y_{SS}}{\frac{R_{SS}}{P_{SS}}}$	(59)	$Y_{SS} = C_{SS} + I_{SS}$	(63)
$1 = \beta\left(1 - \delta + \frac{R_{SS}}{P_{SS}}\right)$	(57)	$L_{SS} = (1-\alpha).\frac{Y_{SS}}{\frac{W_{SS}}{P_{SS}}}$	(60)		
$I_{SS} = \delta K_{SS}$	(58)	$Y_{SS} = K_{SS}^{\alpha}.L_{SS}^{1-\alpha}$	(61)		
		$P_{SS} = \left(\frac{W_{SS}}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_{SS}}{\alpha}\right)^{\alpha}$	(62)		

Source: Adapted from Costa Jr. (2016, pp. 42-43)

Now, we will use equation (57) in order to find R_{SS}:

$$R_{SS} = P_{SS}\left[\left(\frac{1}{\beta}\right) - (1-\delta)\right] \tag{64}$$

Note that R_{SS} is a function of only the normalized general price level parameters, therefore its value can be determined. Now, it simply remains to find the steady state of the wage level W_{SS}.

From (62):

$$W_{SS}^{1-\alpha} = P_{SS}(1-\alpha)^{1-\alpha}\left(\frac{\alpha}{R_{SS}}\right)^{\alpha}$$

$$W_{SS} = (1-\alpha)P_{SS}^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{R_{SS}}\right)^{\frac{\alpha}{1-\alpha}} \tag{65}$$

Now, in order to satisfy the equilibrium condition, we must determine C_{SS} and I_{SS}, that is, the variables of aggregate demand. The idea underlying the notion of equilibrium in these models is a condition of market adjustment. Formally speaking, it means that, given k markets, if demand equals supply in $k-1$ markets and $P_k > 0$, then demand must be equal to supply in the k^{th} market. Otherwise, Walras' Law would be violated (Costa Jr., 2016).

Thus, finding the equilibrium condition means meeting the input market conditions, that is, finding the equilibrium between supplies and demands of labor and capital.

To this end, we first need to substitute (60) in equation (56) and to solve for C_{SS}:

$$C_{SS}{}^{\sigma}\left[(1-\alpha).\frac{Y_{SS}}{\frac{W_{SS}}{P_{SS}}}\right]^{\varphi}=\frac{W_{SS}}{P_{SS}}$$

$$C_{SS}=\frac{1}{Y_{SS}{}^{\frac{\varphi}{\sigma}}}\left[\frac{W_{SS}}{P_{SS}}\left(\frac{\frac{W_{SS}}{P_{SS}}}{1-\alpha}\right)^{\varphi}\right]^{\frac{1}{\sigma}} \tag{66}$$

To find I_{SS}, we need to replace (59) in equation (58):

$$I_{SS}=\left(\frac{\delta\alpha}{R_{SS}}\right).Y_{SS} \tag{67}$$

Finally, substituting (66) and (67) into expression (63), we find Y_{SS}:

$$Y_{SS}=\frac{1}{Y_{SS}{}^{\frac{\varphi}{\sigma}}}\left[\frac{W_{SS}}{P_{SS}}\left(\frac{\frac{W_{SS}}{P_{SS}}}{1-\alpha}\right)^{\varphi}\right]^{\frac{1}{\sigma}}+\left(\frac{\delta\alpha}{R_{SS}}\right).Y_{SS}$$

$$\left(1-\frac{\delta\alpha}{R_{SS}}\right).Y_{SS}=\frac{1}{Y_{SS}{}^{\frac{\varphi}{\sigma}}}\left[\frac{W_{SS}}{P_{SS}}\left(\frac{\frac{W_{SS}}{P_{SS}}}{1-\alpha}\right)^{\varphi}\right]^{\frac{1}{\sigma}}$$

$$Y_{SS} = \left(\frac{R_{SS}}{R_{SS} - \delta\alpha}\right)^{\frac{\sigma}{\sigma+\varphi}} \left[\frac{W_{SS}}{P_{SS}} \left(\frac{\frac{W_{SS}}{P_{SS}}}{1-\alpha}\right)^{\varphi}\right]^{\frac{1}{\sigma+\varphi}} \tag{68}$$

With the equations determined at the steady state and using the calibrated data shown in Table 3, we arrive at the steady state values for the variables (Table 4).

Table 3 - Calibrated Values of the Structural Model

Parameter	Meaning	Calibrated Value
$\boldsymbol{\sigma}$	Relative risk aversion coefficient	2
$\boldsymbol{\varphi}$	Marginal disutility of labor supply	1.5
$\boldsymbol{\alpha}$	Elasticity of capital production	0.35
$\boldsymbol{\beta}$	Discount factor	0.985
$\boldsymbol{\delta}$	Depreciation rate	0.025
$\boldsymbol{\rho_A}$	Autoregressive parameter of productivity	0.95
$\boldsymbol{\sigma_A}$	Standard deviation of productivity	0.01

Adapted from Costa Jr. (2016, p. 47).

One last feature regarding RBC models, which – in fact – extends to all non-linear models in general, is that handling and solving them can be very grueling. Contrariwise, linear models are often easier to handle. So, one solution would be converting a non-linear model to a sufficiently appropriate linear approximation, such that its resolution helps to understand the behavior of the underlying non-linear system. The standard procedure used by economists is the log-linearization around the model's steady state[22].

[22] Costa Jr. (2016, pp. 48-52) shows a method of log-linearization, which is called the Uhlig's method. It is worth saying that some software (e.g., Dynare) can solve DSGE models without the need of linearization. We will not present the linearization here. For the formalization and analysis of impulse-response functions of the presented model, see Costa Jr. (2016, pp. 48-57).

Table 4 - Values of Variables at the Steady State

Variable	Steady State Value
A	1
R	0.040
W	2.084
Y	2.338
I	0.508
C	1.829
L	0.729
K	20.338

Source: Costa Jr. (2016, p. 47)

Now that we have presented the formalization and the main assumptions of a basic RBC model, we want to summarize these types of models with six characteristics, following Carlin and Soskice (2015, pp. 589-590):

a) There is a large number of identical agents in the economy;
b) Agents are assumed to live forever and each one is referred to as "representative agent";
c) It is a world with perfect competition and perfect information;
d) Expectations are formed rationally;
e) There is total flexibility of nominal wages and prices. Thus, one shall work entirely in real terms;
f) The economy may only be disturbed by technology shocks, which have inbuilt persistence, that is, they die out gradually over time.

2.4 NK-Models

2.4.1 Introduction

Based on Keynesian principles, NK theory embraces the idea that economies are subject to market failures, which generate fluctuations. On the contrary, RBC theory states that fluctuations are natural and efficient responses to the technological state of an economy. Thus, for the RBC approach, the aggregate economy is in perfect competition on the demand and on the supply sides (Carlin and Soskice, 2015).

An important consequence is that governments have no active role in the macroeconomics of RBCs, while in the NK approach governments may have an important role in improving welfare.

According to Carlin and Soskice (2015, p. 609):

> When the economy is disturbed by shocks, the cycles that arise in the RBC model are equilibrium cycles. They are also efficient cycles because they represent first best outcomes: the labor market always clears. There is no role for a policy maker to improve welfare. In the NK model, a second kind of imperfection in addition to monopoly power in the goods market, named rigidity, keeps the economy away from the flexible price equilibrium in a way that could be improved on by the intervention of a policy maker. This explains the welfare-enhancing role of a central bank in this model and the rationale for a Taylor rule. Nevertheless, these cycles are still equilibrium cycles as they result from forward-looking best-response decision making by all parties. […] Using a Taylor rule, the forward-looking central bank can improve welfare by steering the economy back to the flexible price equilibrium at least welfare cost.

The basic RBC model presented before is fully based on the assumption of perfect competition in both goods and inputs markets. But if the structure of perfect competition is removed, what happens? The answer to this question is what NK-models try to give us, i. e., with the introduction of imperfect competition – that is the heart of the NK-modelling – how would the economy behave?

For this basic NK-model, we will maintain the same structure of households behavior, but we will make some significant alterations to the structure of the production sector[23].

2.4.2 Theoretical Structure behind NK-Models

This section presents the ideas of imperfect competition and price rigidity, which are the key-concepts to properly understand the construction and formalization of these types of models

[23] This kind of model was initially developed by Rotemberg (1982), Blanchard and Kiyotaki (1987), Rotemberg and Woodford (1997), and others.

2.4.2.1 Differentiated Products and the Consumption Aggregator

In the real world, consumers buy a large number of goods and services that give them "utility". NK-models consider that there is a large number of consumption options available in the market, with slightly difference between them. One way to adapt the use of a single consumption good is assuming that everything is made up of these many differentiated products.

Formally, it is assumed that consumption is a function of N different products:

$$c = c(c_1, c_2, \dots, c_N),$$

where c_1 is a type 1 consumer good, c_2 is a type 2 consumer good and so on. Thus, having N different products, the total consumption will be a function of N different types of consumer goods, formally known as a *consumption aggregator function*, which must satisfy two properties:

$$\frac{\partial c(\cdot)}{\partial c_j} > 0$$

$$\frac{\partial^2 c(\cdot)}{\partial c j^2} < 0$$

The first represents that the total consumption is an increasing function of a j type good. The second one means that the total consumption function increases at diminishing rates.

The aggregate consumption function most used in NK-models is a CES (Constant Elasticity of Substitution):

$$c(c_1, c_2, \dots, c_N) = \left[(c_1)^{\frac{\psi-1}{\psi}} + (c_2)^{\frac{\psi-1}{\psi}} + \cdots + (c_N)^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}$$

In this expression, ψ is the elasticity of substitution between the differentiated goods.[24] Note that, for an aggregate function with two goods only, that is, $c(c_1, c_2) = \left[(c_1)^{\frac{\psi-1}{\psi}} + (c_2)^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}$, the elasticity of substitution measures the proportional change in c_1/c_2 in relation to the proportional change in the marginal rate of substitution (MRS) along an indifference curve. That is:

$$\psi = \frac{\%\Delta(c_1/c_2)}{\%\Delta MRS} = \frac{\partial(c_1/c_2)}{\partial MRS} . \frac{MRS}{(c_1/c_2)} = \frac{\partial \ln(c_1/c_2)}{\partial \ln MRS}$$

2.4.2.2 Monopolistic Competition

The core idea of NK-models lies in the assumption that each of the N differentiated products is presumed to be crafted by different firms, which are in monopolistic competition.

The theoretical idea of monopolistic competition is that all goods are, to some degree, imperfect substitutes for one another. In the economic literature, monopolistic competition is an intermediate theoretical basis between pure monopoly and perfect competition.

From RBC model's assumptions, we saw that – in a perfect competition framework – firms are price-takers, in the sense that they do not have the power to decide the price of their goods and because there is perfect substitutability among all products in the economy. On the contrary, in monopolistic competition, firms often define the price of their goods.

[24]This parameter has great economic significance in NK-models, as it determines to what degree, from a consumer's point of view, products differ from one another. NK-models usually assume $\frac{\omega}{\omega-1} > 1$, that is the goods are imperfect substitutes.

2.4.2.3 Price Stickiness

One of the main challenges of NK theory is to demonstrate how wages and prices stickiness can result from the behavior of optimizing agents. Generally, the following stylized facts about changes in prices and wages are considered by the literature:

a) Prices and wages are rigid for some period of time;
b) Prices and wages are readjusted two or three times a year, on average;
c) High inflation are frequently caused by prices and wages being adjusted frequently;
d) Prices and wages are not adjusted at the same time;
e) Changes in prices of tradable goods occur more frequently than with non-tradable goods.

Thus, the concept of price stickiness refers to a situation where the price of a good does not change readily to a new market-clearing price (equilibrium price) when there are shifts in the demand and/or the supply curves.

2.4.3 A Basic NK-Model[25]

To develop this basic NK-model, we have to introduce price stickiness and monopolistic competition. For this purpose, we will maintain the assumptions that we are facing a closed-economy and that there is no currency in the economy[26].

2.4.3.1 Households

Once again, we will assume a representative infinitely-lived household, seeking to maximize:

[25] We will follow the basic NK-model proposed in Gali (2008).
[26] For our purposes of formalization, we are maintaining these assumptions. However, more complicated DSGE models try to add currency and to deal with an open-economy.

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t N_t) \tag{69}$$

In (69), N_t denotes hours of work or employment and C_t is a consumption index given by:

$$C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\varepsilon}}. di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{70}$$

with $C_t(i)$ representing the quantity of good i consumed by the household in period t.

The period utility $U(C_t N_t)$ is assumed to be continuous and twice differentiable, with:

$$U_{c,t} \equiv \frac{\partial U(C_t N_t)}{\partial C_t} > 0$$
$$U_{cc,t} \equiv \frac{\partial^2 U(C_t N_t)}{\partial C_t^2} \leq 0$$
$$U_{n,t} \equiv \frac{\partial U(C_t N_t)}{\partial N_t} \leq 0$$
$$U_{nn,t} \equiv \frac{\partial^2 U(C_t N_t)}{\partial N_t^2} \leq 0$$

In words, it means that the marginal utility of consumption $U_{c,t}$ is assumed to be positive and nonincreasing, whilst the marginal disutility of labor,$-U_{n,t}$, is positive and nondecreasing.

Assume the existence of a continuum of goods represented by the interval [0,1]. The maximization of (69) faces a sequence of flow budget constraints given by:

$$\int_0^1 P_t(i) C_t(i). di + Q_t B_t \leq B_{t-1} + W_t N_t + T_t \tag{71}$$

for $t = 0,1,2, \ldots$, where $P_t(i)$ is the price of good; W_t is the nominal wage; B_t represents the quantity of one-period, nominally riskless, discount bonds acquired in period t and maturing in period $t+1$; Q_t is the price paid for each bond at maturity; T_t represents lump-sum additions or subtractions to period income, expressed in nominal terms. N_t is the hours of work (or the measure of household members employed). P_t, W_t and Q_t are assumed to take as given.

In addition to (71), we will consider a solvency constraint of the form:

$$\lim_{T\to\infty} E_t(B_t) \geq 0 \tag{72}$$

Condition (72) prevents household from engaging in Ponzi-type schemes.

Now, with differentiated goods, household must decide how to allocate its consumption expenditures. This requires the maximization of C_t for any given level of expenditures $\int_0^1 P_t(i)C_t(i).di$.

As shown in Gali, (2008, p. 61), the solution to that problem yields the set of demand equations:

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon}.C_t \tag{73}$$

for all $i \in [0,1]$, where $P_t \equiv \left[\int_0^1 P_t(i)^{1-\varepsilon}.di\right]^{\frac{1}{1-\varepsilon}}$ is an aggregate price index.

Conditional on this optimal behavior, total consumption expenditures can be written as the product of the price index times the quantity index, i.e.:

$$\int_0^1 P_t(i)C_t(i).di = P_tC_t \tag{74}$$

Plugging (74) into (71) yields:

$$P_tC_t + Q_tB_t \leq B_{t-1} + W_tN_t + T_t \tag{75}$$

The optimality conditions implied by the maximization of (69) subject to (75) are given by[27]:

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \tag{76}$$

$$Q_t = \beta . E_t \left[\frac{U_{c,t+1}}{U_{c,t}} . \frac{P_t}{P_{t+1}} \right] \tag{77}$$

Note that this model is assuming a period utility given by:

$$U(C_t N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \tag{78}$$

Thus, equation (76) can be rewritten in log-linear form as:

$$w_t - p_t = \sigma . c_t + \varphi . n_t \tag{79}$$

where lower case letters denote the natural logs of the corresponding variable (i.e., $c_t \equiv \log C_t$). Note that (79) can be interpreted as a competitive labor supply schedule, which determines the quantity of labor supplied as a function of the real wage, given the marginal utility of consumption (which, under the assumptions, is a function of consumption only).

A log-linear approximation of (77) around the steady state, with constant rates of inflation and consumption growth, yields the log-linearized Euler equation:

$$c_t = E_t(c_{t+1}) - \frac{1}{\sigma}(i_t - E_t(\pi_{t+1}) - \rho \tag{80}$$

where $i_t \equiv -\log Q_t$; $\rho \equiv -\log \beta$ and $\pi_{t+1} \equiv p_{t+1} - p_t$ is the rate of inflation between t and $t+1$, with $p_t \equiv \log P_t$.

Notice that i_t corresponds to the log of the gross yield on the one-period bond, that is, the nominal interest rate. Following the same logic, ρ can be interpreted as the discount rate of household.

[27] For the sake of briefness, we will skip the resolution of this maximization problem, but it is properly demonstrated in Gali (2008, p. 17).

2.4.3.2 Firms

Consider a continuum of firms indexed by $i \in [0,1]$. Also, assume that each firm produces a differentiated good, using the same technology. The production function is given by:

$$Y_t(i) = A_t . N_t(i)^{1-\alpha} \tag{81}$$

where A_t represents the level of technology, which is identical to all firms and evolves exogenously over time.

Assume that all firms decide how much to produce in each period following a Calvo's rule (Calvo, 1983), i. e., independent of the time elapsed since the last adjustment, each firm can reset its price only with probability $1-\theta$ in any given period, whilst θ firms maintain their prices unchanged. As a result, the average duration of a price is given by $(1-\theta)^{-1}$. Note that in this context θ is considered a natural index of price stickiness.

2.4.3.2.1 Aggregate Price Level Dynamics

Let $S(t) \subset [0,1]$ represent the set of firms not reoptimizing prices in period t. Using the definition of the aggregate price level (see equation (73)) and considering the fact that all firms choose an identical price P_t^* when resetting their prices[28]:

$$P_t = \left[\int_{S(t)} P_{t-1}(i)^{1-\varepsilon} . di + (1-\theta)(P_t^*)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

$$P_t = [\theta(P_{t-1})^{1-\varepsilon} + (1-\theta)(P_t^*)^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} \tag{82}$$

where (82) comes from the fact that the distribution of prices among firms not adjusting in period t corresponds to the distribution of effective prices in period $t-1$, but with total mass reduced to θ.

[28] All firms choose the same price because they face an identical problem.

Dividing both sides of (82) by P_{t-1}:

$$\pi_t^{1-\varepsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon} \tag{83}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$. Notice that in a steady state with zero inflation $P_t^* = P_{t-1} = P_t$, for all t.

Finally, log-linearizing (83) around $\pi_t = 1$ and $\frac{P_t^*}{P_{t-1}} = 1$ yields:

$$\pi_t = (1-\theta)(\, p_t^* - p_{t-1}) \tag{8456}$$

Equation (84) shows that, in this model, inflation occurs when firms reoptimize and choose a price that differs from the economy's average price in the previous period. In order to understand the evolution of inflation over time, we need to analyze the factors underlying the price-setting decisions of the firms.

2.4.3.2.2 Optimal Price Setting

A firm reoptimizing in period t will choose the price P_t^* that maximizes the current market value of the profits generated while that price remains effective. The maximization problem is:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t\left[Q_{t,t+k}(P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}))\right]$$

subject to

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{\varepsilon} . C_{t+k} \tag{85}$$

for $k = 0,1,2,\ldots$, where $Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} . \left(\frac{P_t}{P_{t+k}}\right)$ is the stochastic discount factor for nominal payoffs; $\Psi(\cdot)$ is the cost function, and $Y_{t+k|t}$ denotes output in period $t+k$ for a firm that last reset its price in period t.

The first-order condition for (85) is:

$$\sum_{k=0}^{\infty}\theta^k E_t\left[Q_{t,t+k}Y_{t+k|t}(P_t^* - \Phi\psi_{t+k|t})\right] = 0 \qquad (86)$$

where $\psi_{t+k|t} \equiv \Psi_{t+k}^{'}(Y_{t+k|t})$ denotes the (nominal) marginal cost in period $t + k$ for any firm which last reset its price in period t and $\Phi \equiv \frac{\varepsilon}{\varepsilon-1}$.

Note that when $\theta = 0$, that is, when there are no price rigidities, (86) turns into the optimal price-setting condition under flexible prices $P_t^* = \Phi\psi_{t|t}$, which allows us to interpret Φ as the desired markup when there are no constraints on the frequency of price adjustment.

The following step is to linearize (86) around the zero inflation steady state. However, before doing so, it is useful to rewrite it in terms of variables that have a well-defined value in that steady state. Letting $\pi_{t,t+k} \equiv \frac{P_{t+k}}{P_t}$ and dividing by P_{t-1}, equation (86) becomes:

$$\sum_{k=0}^{\infty}\theta^k E_t\left[Q_{t,t+k}Y_{t+k|t}\left(\frac{P_t^*}{P_{t-1}} - \Phi \mathrm{MC}_{t+k|t}\pi_{t-1,t+k}\right)\right] = 0 \qquad (87)$$

where $\mathrm{MC}_{t+k|t} \equiv \frac{\psi_{t+k|t}}{P_{t+k}}$ is the real marginal cost in period $t + k$ for a firm that last set its price in period t.

In the zero inflation steady state, $\frac{P_t^*}{P_{t-1}} = 1$ and $\pi_{t-1,t+k} = 1$. Moreover, constancy of the price level implies that $P_t^* = P_{t+k}$ in that steady state, from which it follows that $Y_{t+k|t} = Y$ and $\mathrm{MC}_{t+k|t} = MC$, because each firm will be producing the same quantity of output. In addition, $Q_{t,t+k} = \beta^k$ must hold in that steady state. Thus, $MC = 1/\Phi$.

A first-order Taylor expansion of (87) around the zero inflation steady state yields:

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t\left[\widehat{mc}_{t+k|t} + (p_{t+k} - p_{t-1})\right] \qquad (8857)$$

where $\widehat{mc}_{t+k|t} \equiv mc_{t+k|t} - mc$ denotes the log deviation of marginal cost from its steady state value $mc = -\mu$, and where $\mu \equiv \log \Phi$ is the log of the desired gross markup (which, for $\Phi \to 1$ is approximately equal to the net markup $\Phi - 1$).

Let us rewrite (88) in a way that we can obtain more intuition about the factors determining a firm's price-setting decision.

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t\left(mc_{t+k|t} + p_{t+k}\right) \tag{89}$$

Notice from (89) that firms, when resetting their prices, will choose a price that corresponds to their desired markup over a weighted average of their current and expected (nominal) marginal costs, with the weights being proportional to the probability of the price remaining effective at each horizon θ^k.

2.4.3.2.3 Equilibrium

Market clearing in the goods market implies:

$$Y_t(i) = C_t(i) \tag{90}$$

for all $i \in [0,1]$ and all t.

Defining aggregate output as $Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\varepsilon}}.di\right)^{\frac{\varepsilon}{\varepsilon-1}}$ it follows that:

$$Y_t = C_t \tag{91}$$

Equation (91) must hold for all t.

Combining the Euler equation (80) with the above goods market clearing condition (91) yields the equilibrium condition:

$$y_t = E_t(y_{t+1}) - \frac{1}{\sigma}(i_t - E_t(\pi_{t+1}) - \rho \tag{92}$$

Market clearing in the labor market requires:

$$N_t = \int_0^1 N_t(i).di \tag{93}$$

Using (81):

$$N_t = \int_0^1 \left(\frac{Y_t(i)}{A_t}\right)^{\frac{1}{1-\alpha}}.di \tag{94}$$

From (73) and (90):

$$N_t = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}}.di \tag{95}$$

Taking logs:

$$(1-\alpha)n_t = y_t - a_t + d_t \tag{96}$$

where $d_t \equiv (1-\alpha).\log\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}}$ and di is a measure of price dispersion across firms (and, hence, output).

In a neighborhood of the zero inflation steady state, d_t is equal to zero up to a first-order approximation. Thus, (96) becomes:

$$y_t = a_t + (1-\alpha)n_t \tag{97}$$

We can interpret (97) as an approximated relation between aggregate output, employment and technology.

Now, let us derive an individual firm's marginal cost in terms of the economy's average real marginal cost:

$$mc_t = (w_t - p_t) - mpn_t$$
$$mc_t = (w_t - p_t) - (a_t - \alpha n_t) - \log(1-\alpha)$$
$$mc_t = (w_t - p_t) - \frac{1}{1-\alpha}(a_t - \alpha y_t) - \log(1-\alpha) \tag{98}$$

for all t. Note that we defined the economy's average marginal product of labor $mpnt$ in a way consistent with (97).

Knowing that:

$$mc_{t+k|t} = (w_{t+k} - p_{t+k}) - mpn_{t+k|t}$$
$$mc_{t+k|t} = (w_{t+k} - p_{t+k}) - \frac{1}{1-\alpha}\left(a_{t+k} - \alpha y_{t+k|t}\right) - \log(1-\alpha)$$

then:

$$mc_{t+k|t} = mc_{t+k} + \frac{\alpha}{1-\alpha}\left(y_{t+k|t} - y_{t+k}\right)$$

Using the demand schedule (73) combined with the market clearing condition (91), we get:

$$mc_{t+k|t} = mc_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}(p_t^* - p_{t+k}) \tag{99}$$

Notice that assuming $\alpha = 0$ (constant returns to scale) $mc_{t+k|t} = mc_{t+k}$, i.e., marginal cost becomes independent of the level of production and, hence, is common across firms.

Substituting (99) into (88) and rearranging terms yields:

$$p_t^* - p_{t-1} = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k E_t[\Theta\,\widehat{mc}_{t+k} + (p_{t+k} - p_{t-1})]$$
$$p_t^* - p_{t-1} = (1-\beta\theta).\Theta\sum_{k=0}^{\infty}(\beta\theta)^k E_t(\widehat{mc}_{t+k}) + \sum_{k=0}^{\infty}(\beta\theta)^k\, E_t(\pi_{t+k}) \tag{100}$$

with $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\varepsilon} \leq 1$.

Equation (100) can be rewritten in a more compacted way, as follows:

$$p_t^* - p_{t-1} = \beta\theta. E_t[p_{t+1}^* - p_t] + (1-\beta\theta). \Theta. \widehat{mc}_t + \pi_t \quad (101)$$

Finally, combining (101) and (84) yields the inflation equation:

$$\pi_t = \beta. E_t[\pi_{t+1}] + \lambda\widehat{mc}_t \quad (102)$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\Theta$ is strictly decreasing in the index of price stickiness θ, in the measure of returns to scale α, and in the elasticity of demand ε.

Let us express (102) as the discounted sum of current and expected future deviations of real marginal costs from the steady state, that is:

$$\pi_t = \lambda \sum_{k=0}^{\infty} \beta^k E_t(\widehat{mc}_{t+k}) \quad (103)$$

Defining the economy's average markup as $\mu_t = -mc_t$, we can see that inflation π_t will be higher when firms expect average markups to be below their steady state (i.e., desired) level μ, for in that case firms that have the chance to reset their prices will choose a price above the average price of the economy in order to realign their markup closer to its desired level.

Notice that the mechanism underlying fluctuations in the aggregate price level and inflation in this model is a consequence of purposeful price-setting decisions by firms, which adjust their prices according to a current and anticipated cost conditions.

Now, we will derive a relation between the economy's real marginal cost and a measure of aggregate economic activity by using the household's optimally condition (79) and the aggregate production relation (97). Notice that regardless of the nature of price setting, average real marginal cost can be expressed as:

$$mc_t = (w_t - p_t) - mpn_t$$
$$mc_t = (\sigma y_t + \varphi n_t) - (y_t - n_t) - \log(1-\alpha)$$
$$mc_t = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t - \frac{1+\varphi}{1-\alpha} a_t - \log(1-\alpha) \tag{104}$$

As shown at the end of section 2.4.3.2.2, under flexible prices the real marginal cost turns out to be constant and given by $mc = -\mu$. Defining the natural level of output, y_t^n, as the equilibrium level of output under flexible prices:

$$mc = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n - \frac{1+\varphi}{1-\alpha} a_t - \log(1-\alpha) \tag{105}$$

thus implying:

$$y_t^n = \psi_{ya}^n + \vartheta_y^n \tag{106}$$

where $\vartheta_y^n \equiv -\frac{(1-\alpha)(\mu - \log(1-\alpha))}{\sigma(1-\alpha)+\varphi+\alpha} > 0$ and $\psi_{ya}^n \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$.

It is important to notice that the presence of market power by firms has the effect of lowering the output level uniformly over time, without affecting its sensitivity to shifts in technology.

Now, subtracting (105) from (104):

$$\widehat{mc}_t = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)(y_t - y_t^n) \tag{107}$$

Expression (107) means that the log deviation of real marginal cost from steady state is proportional to the log deviation of output from its flexible price counterpart. That deviation is referred in the literature as the *output gap*, and is denoted by $\tilde{y}_t \equiv y_t - y_t^n$.

One of the key building blocks of the basic NK-model can be obtained by combining (107) with (102):

$$\pi_t = \beta E_t[\pi_{t+1}] + \kappa\, \tilde{y}_t \tag{108}$$

where $\kappa \equiv \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$

Equation (108) relates inflation to its one period ahead forecast and the output gap. It is referred to as the *New Keynesian Phillips Curve (NKPC)* in the literature.

A second key equation describing the equilibrium of the model is the *dynamic IS equation (DIS).* It can be obtained by rewriting (92) in terms of the output gap:

$$\tilde{y}_t = -\frac{1}{\sigma}(i_t - E_t[\pi_{t+1}] - r_t^n) + E_t[\tilde{y}_{t+1}] \tag{109}$$

where r_t^n is the *natural rate of interest*, described as:

$$r_t^n \equiv \rho + \sigma\, E_t[\Delta y_{t+1}^n]$$
$$r_t^n \equiv \rho + \sigma\, \psi_{ya}^n\, E_t[\Delta a_{t+1}] \tag{110}$$

Assume that the effects of nominal rigidities vanish asymptotically, i.e. $\lim_{T\to\infty} E_t[\pi_{t+1}] = 0$. In this case, resolving equation (109) forward yields the expression:

$$\tilde{y}_t = -\frac{1}{\sigma}\sum_{k=0}^{\infty}(r_{t+k} - r_{t+k}^n) \tag{111}$$

where $r_t \equiv i_t - E_t[\pi_{t+1}]$ is the expected real return on a one period bond, that is, the real interest rate.

Equation (111) makes it clear that the output gap is proportional to the sum of current and anticipated deviations between the real interest rate and its natural counterpart.

Together with an equilibrium process for the natural rate, r_t^n, equations (108) and (109) constitute the *non-policy block* of the NK-model. Notice that this block has a simple iterative structure: the NKPC determines inflation given the path for the output gap, whereas the DIS equation determines the output gap given a path for the (exogenous) natural rate and the actual real rate.

Finally, in order to close the model, one only needs to supplement (108) and (109) with one or more equations determining how the nominal interest rate i_t behaves over time. In other words, a description of how the monetary policy is conducted. The main result of this exercise is that, in the presence of prices rigidity, the equilibrium path of real variables cannot be determined independently of monetary policy, that is, monetary policy is non-neutral

2.5 Final Remarks

The models presented so far, albeit convenient for our illustrative-analytical demonstrations, cannot be considered rich enough to be used for many applications. Any policymaker wanting to forecast the long-run path of the economy or evaluate the macroeconomic effects of some policy intervention would certainly need considerably more complicated models.

There is a large and active literature engaged in building and estimating more sophisticated quantitative models. Although these models are much more complicated and their analytical treatability is almost unmanageable, at their core, they have important resemblances to the models of the previous sections. Broadly speaking, they implement important modifications and extensions to the baseline model discussed in section 2.4.3, especially regarding aggregate demand, aggregate supply, credit-market imperfections and policy assumptions.[29]

For a last observation, it is worth saying that almost all macroeconomists agree that models have important strengths and weaknesses. The use of models for macroeconomic evaluation often fall along a continuum between two extremes (although just a few economists actually are at either extreme):

> One extreme is that we are well on the way to having models of the macroeconomy that are sufficiently well grounded in microeconomic assumptions that their parameters can be thought of as structural (in the sense that they do not change when policies change), and that are sufficiently realistic that they can be used to obtain welfare-based recommendations about the conduct of policy. Advocates of this view can point to the facts that the models are built up from microeconomic foundations; that estimated versions of the models match some important features of fluctuations reasonably well; that many policymakers value the models enough to put weight on their predictions and recommendations; that there is microeconomic evidence for many of their assumptions; and that their sophistication is advancing rapidly.
> The other extreme is that the models are ad hoc constructions that are sufficiently distant from reality that their policy recommendations are unreliable and their predictions likely to fail if the macroeconomic environment changes. Advocates of this view can point to two main facts. First, despite the models' complications, there is a great deal they leave out. For example, until the recent crisis, the models' treatment of credit-market

[29] See Romer (2012, pp. 356-360) for a briefly discussion about these extensions.

> imperfections was generally minimal. Second, the microeconomic case for some important features of the models is questionable. Most notably, the models include assumptions that generate inertia in decision making: inflation indexation in price adjustment, habit formation in consumption, and adjustment costs in investment. The inclusion of these features is mainly motivated not by microeconomic evidence, but by a desire to match macroeconomic facts. For example, at the microeconomic level we see nominal prices that are fixed for extended periods, not frequently adjusted to reflect recent inflation. Similarly, standard models of investment motivated by microeconomic evidence involve costs of adjusting the capital stock, not costs of adjusting investment. The need to introduce these features, in this view, suggests that the models have significant gaps." (Romer, 2012, pp. 360-361)

The truth lies somewhere between the two extremes. Therefore, it seems important to understand different approaches. Exploiting diversity in macroeconomic modeling starts with the investigation into new possibilities that are emerging in the literature. This is what the next chapter intend to do.

3. UNDERSTANDING ABMs

Not all the economists think that DSGE models are the only way to proceed macroeconomic analysis. In fact, there is a growing collection of papers and books trying to set out an alternative approach to macroeconomics without making the restrictive assumptions found in the DSGE models. In particular, this alternative approach relies on the complexity of macroeconomics, that is, the idea that macroeconomics is a complex-dynamical system of heterogeneous agents - endowed with bounded rationality and limited (i. e., incomplete) information set - that interact directly and indirectly with each other and the environment. The advocates of this approach believe that macroeconomic theory "should be explained as emerging from the continuous adaptive dispersed interactions of a multitude of autonomous, heterogeneous and bounded rational agents living in a truly uncertain environment" (Delli Gatti *et. al*., 2011). For this end, ABMs would be the best methodological tool.

It should be noted that ABMs have been employed in a number of fields, such as the building of an artificial stock exchanges, industrial dynamics, environmental regulation and the analysis of the effects of macroeconomic policies on output, employment and economic growth. In what follows, we focus our attention on the subset of ABMs evaluating the impact of macroeconomic policies, which can be straightforwardly compared to DSGE models and can respond to the new theoretical and empirical challenges raised by the Great Recession. Following the structure of Fagiolo and Roventini (2017), we will make a brief foray into ABMs' issues classifying them in four areas: fiscal policy, monetary policy, macroprudential policy and labor market policy.

Fiscal Policy

The crisis of 2008 has reawaked interest for employing fiscal policies to tackle economic downturns. Dosi et al. (2010) try to study both the short and long-run impact of fiscal policies by developing an ABM that bridges Keynesian theories of demand-generation and Schumpeterian theories of technology-fueled economic growth (the K+S model) [30]. In the full-fledge version, the K+S model is populated by heterogeneous

[30] See Dosi et al. (2016b) for a survey.

capital-good firms, consumption good-firms, consumers/workers, banks, Central Bank, and a public sector. Capital-good firms perform R&D and sell heterogeneous machine tools to consumption-good firms. Consumers supply labor to firms and fully consume their received income. Banks provide credit to consumption-good firms, financing their production and investment decisions. The Central Bank fixes the short-run interest rate and the government levies taxes. According to Fagiolo and Roventini (2016), the model of Dosi et al. (2010) can endogenously generate growth and jointly account for mild recessions and deep downturns. Moreover, it can also replicate an ensemble of stylized facts concerning both macroeconomic dynamics (e.g. cross-correlations, relative volatilities, output distributions) and microeconomic ones (firm size distributions, firm productivity dynamics, firm investment patterns).

After having been empirically validated according to the output generated, the K+S model is employed to study the impact of fiscal policies (i.e., tax rate and unemployment benefits) on average GDP growth rate, output volatility and unemployment rate. The authors find that Keynesian fiscal policies are a necessary condition for economic growth and they can be successfully employed to reduce economic fluctuations. Furthermore, Dosi et al. (2013) find a strong interaction between income distribution and fiscal policies: the more income distribution is slanted toward profits, the greater the case for fiscal policies to dampen macroeconomic fragility (Fagiolo and Roventini, 2012, 2016).

In Dosi et al. (2015), the authors study different fiscal austerity policies and they find that fiscal consolidation rules are "self-defeating", as they depress the economy without improving public finances (similar conclusions are reached by Teglio et al. (2015)) employing the EURACE model (Cincotti et al., 2010, 2012). Moreover, the negative effects of fiscal policies are magnified by higher level of income inequality (Dosi et al., 2015). Finally, austerity policies can also reduce long-run productivity and GDP growth, by harming innovation rate and the diffusion of new technologies (Dosi et al., 2016b) and firms' investment rates (Bassi and Lang, 2016). In fact, stabilization policies can affect both short and long-run dynamics as found also by Russo et al. (2007) and Harting (2015).

Many ABMs explore the interactions between financial instability and fiscal policies, as in Napoletano et al. (2015), where they build an ABM populated by heterogeneous households facing time-varying credit constraints. What they find is that deficit-spending fiscal policy dampens the magnitude and persistence of bankruptcy shocks. Also, the size of the multipliers changes over time and it is related to the evolution of credit rationing.

Chiarella and Di Guilmi (2012) explore the consequences of financial fragility from the firms' perspective building an ABM in which the investment of heterogeneous firms is conditioned by market expectations. Moreover, money can be either exogenous or endogenous and the Government can levy taxes on profits or private wealth. The model shows that with endogenous money and credit, a wealth tax is a more effective stabilization policy than a tax on profit. In the same line, in an ABM with heterogeneous workers, firms, and banks interacting in markets through a decentralized matching protocol, Riccetti et al. (2014) find that during extend crises triggered by bank defaults and financial instability, the Government can stabilize the economy.

In Dosi et al. (2016a), they study the impact of different expectation-formation mechanisms in the K+S model. They start from the Brock and Hommes (1997) framework and find that austerity policies are self-defeating even when agents can switch among different expectation rules (e.g. adaptive, trend-follower expectations) as in Anufriev et al. (2013). Moreover, in line with Gigerenzer (2007) and Gigerenzer and Brighton (2009), they find that the performance of the economy does not improve when agents are more rational. On the other hand, when agents employ Ordinary Least Square (OLS) to form their forecasts, the individual and collective performance worsen as structural breaks and Knightian uncertainty cannot be taken into account (Fagiolo and Roventini, 2016). Relatedly, Haber (2008) studies the interactions between different expectation and formation mechanisms with fiscal and monetary policies in an ABM. He finds out that the introduction of more classy expectations reduces the effects of fiscal policy, whereas it increases the impact of monetary policy.

Monetary Policy

The financial crisis of 2008 showed the importance of the financial market in destabilizing the economy and that monetary policy alone is not capable of putting the economy back on a steady growth path. Thus, in the last ten years, a number of projects have been initiated that aim at the development of closed macroeconomic models using an agent-based approach. In general, these models share the description of macroeconomic dynamics as the outcome of aggregation of interaction on the micro-level, but their focus varies quite significantly. According to Dawid et al. (2013), so far there is no general standard agent-based macroeconomic framework.

According to Fagiolo and Roventini (2017), an increasing set of ABMs are employing Taylor rules to explore the effects of monetary policy on the economy. In this respect, such policy analyses exercises are like the ones conducted with DSGE models, but the complexity-rooted approach of ABM can bring fresh new insights.

The model of Dosi et al. (2015) assesses the effects of a central bank following the achievement of a given inflation target against a central bank that has a dual goal of controlling inflation and employment. They find that a dual-purpose central bank is more efficient in stabilizing the economy without substantially increasing the inflation rate than a central bank that only pursues an inflation target (similar results were found in Raberto et al. (2008), and Delli Gatti and Desiderio (2015)).

Delli Gatti et al. (2005) study alternative commitment vis-à-vis discretionary monetary strategies in an economy populated by heterogeneous, interacting firms and workers. They find that persistent capital market imperfections imply that monetary policy affects the economy through the credit channel and that money is not neutral in the long-run. Moreover, the Taylor principle does not sustain, and the adaptive rule outperforms the commitment one according to the standard loss function criterion.

Arifovic et al. (2010) study the issue of monetary policy dynamic inconsistency in an ABM with a central bank and heterogeneous agents with bounded rationality. Agents may believe in the inflation announced by the central bank or use an adaptive learning process to shape their inflation expectations. This work shows that the central bank learns to maintain the equilibrium with a floating and positive percentage of agents who believe in its policy and that this result is more efficient than the equilibrium found in the more conventional works. Salle et al. (2013) evaluate the performance of an inflation targeting regime in which heterogeneous agents use heuristics and continuously learn to use genetic algorithm. This paper reveals that credibility in the inflation targeting regime has a central role in achieving monetary policy objectives and that transparency of the inflation target is important to increase credibility in the target regime and stabilize the economy.

Cincotti et al. (2010) explore the effects of unconventional monetary policy. They developed an ABM based on the EURACE platform to assess the effects of quantitative-easing monetary policy. The simulation results show that there is an improvement in the economy’s performance when expansionary fiscal policy and quantitative-easing monetary policy are implemented. However, such expansionary policies raise inflation and lead to higher output volatility in the long-run.

Arifovic and Maschek (2012) consider an open-economy background, where the Central Bank fixes the interest rate trying to avoid currency crisis. They find that decreasing the interest rate under the threat of a possible currency attack is more efficient than defending the currency, as the second policy may increase the outflow of funds.

Financial instability, Bank regulation and Macroprudential policies

Regarding the issue of instability and financial regulation, the work of Ashraf et al. (2011) develops an ABM in which heterogeneous firms interact with banks that provide loans to these firms. This research indicates that in steep recessions bank lending can stabilize the economy and that less restrictive banking regulation allows the economy to recover more quickly. Dosi et al. (2013) show, in a model of Keynesian and Schumpeterian agents that includes banks, that larger loans have positive impacts on growth when firms cannot rely on their own resources. Raberto et al. (2012) employ the EURACE model and find that lower capital-adequacy ratios can stimulate growth in the short-run, whilst, in the long-run, high stocks of private debt can lead to more firm bankruptcies, credit rationing and more serious economic downturns. Klimek et al. (2015) study alternative resolution mechanisms of banking crises and they find that in period of expansions, the best policy to achieve financial and economic stability is closing the distressed bank.

The works of Poledna et al. (2014) and Aymanns and Farmer (2015) show in an ABM that Basel II[31] has destabilizing effects by increasing synchronized buying and selling activities in view of the need to reduce leverage. There is also a new generation of ABMs employed to study the effects of Basel III[32] macroprudential regulation and its

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[31] Basel II is an international regulatory accord from 2004. It is a set of international bank regulations based on three main pillars: minimal capital requirements, regulatory supervision and market discipline. Its main objective was to increase minimal capital requirements established under Basel I, the first international regulatory accord (1988).

[32] Basel III (2009) is part of a continuous effort to enhance the banking regulatory framework. It introduced a set of reforms designed to improve the regulation, supervision and risk management within the banking sector. Its main idea is to foster greater resilience at the individual level in order to reduce the risk of a system-wide shock.

possible interactions with monetary policy to achieve price and financial stability, e.g., Popoyan et al. (2015); Krug et al. (2015);

According to Fagiolo and Roventini (2017), the modeling of a network structure is difficult in DSGE models. As they state, “this lack of consideration has prevented such models to explain the emergence, the depth and the diffusion of the current crisis, where the topological properties of the credit market network have a fundamental” (Fagiolo and Roventini, 2017, p. 21). Taking a complexity theory perspective and combining network theory and ABMs is an interesting work that can help to avoid the occurrence of financial crises (Battiston et al., 2016). Some works regarding this subject are Gai et al. (2011); Krause and Giansante (2012); Gaffeo and Molinari (2016).

Still within the same theme, Gabbi et al. (2015) develops an ABM with network between banks and a real sector to assess the impact of some macroprudential regulations. They find that the effect of regulation on banks' performance changes according to the state of the economy, the degree of network connection between banks and the volume of information about banks' risks.

Labor Market Policy

Fagiolo and Roventini (2017, p. 22) state that “in DSGE models, labor market is not usually modeled, and unemployment is not contemplated. This prevents them to study problems related to involuntary unemployment, structural reforms, human capital policies, etc.”. The model of Dosi et al. (2016d, 2016e, 2016f) extend the K+S model to account for different microfounded labor-market regimes defined by different levels of wage flexibilities, labor mobility and institutions. The model is capable of generating persistent involuntary unemployment and it accounts for several stylized facts of the labor market. The results show that the more rigid labor markets and labor relations are the higher productivity and GDP growth will be, whilst leading to lower inequality, unemployment and output volatility. Similar results are found in Napoletano et al. (2012) and Seppecher (2012).

In Dawid et al. (2014), it is shown that the effects of policies - such as improving workers’ skills and firms’ technological adoption on innovation, commuting flows, inequality dynamics and economic convergence – depends on the flexibility of the labor markets.

As shown above, ABM’s contribution is increasing as the methodology becomes more widespread. Albeit the issues regarding this modelling methodology cannot be neglected, we believe that a more fruitful development of macroeconomics should take into account the basic topics raised by the AB modelers.

That being said, the main aim of this chapter is to understand how ABMs work. For this ultimate achievement, we begin with an introduction that shows the main criticisms raised against the DSGE approach. Following that, the ideas behind the agent-based methodology are presented. Then, the structure of a basic ABM is elaborated. At last, the issues of validation and estimation are discussed.

3.1. Background

The 2008 economic crisis has undoubtedly challenged the mainstream approach to macroeconomic modelling. In fact, a great number of economists argue that the economic crisis has indeed produced a crisis in macroeconomics. One of the main reasons for this apparent failure of the standard approach, grounded into the DSGE models, relies on the assumptions that the economy is somehow capable of reaching and sustaining an equilibrium path.

However, a different view that the economy is a non-linear, complex and dynamic system, which rarely, if ever, reaches equilibrium, may offer a way around. While in a linear system, macro level activity amounts to a simple adding up of the micro actions, in a non-linear system, something new may emerge (Hamill, L. and Gilbert, N., 2016). In a complex system, the whole may constitute something which is more and different than the mere aggregation of its constitutive parts.

The proponents of this methodology argue that an important feature of macroeconomic modelling resides in the ability to analyze evolutionary complex system like the economic one. For this end, ABMs would be the best methodological instrument, as it is appropriate to study complex dynamics as the result of the interaction of heterogeneous agents[33]. The idea that the economy is a complex adaptive system is at the heart of the Agent-Based modelling.

[33] Notice that one can interpret a *representative-agent-model* as a degenerate case in which the degree of both heterogeneity and interaction is set to zero, which is a situation that reduces holism to reductionism in a hypothetical world without networks and coordination problems.

On the other hand, Agent-Based modelers usually argue that the capability of DSGE models to encompass and explain complex events is very weak:

> Mainstream macroeconomic models do not take into consideration that there might be two-way interdependencies between individuals and aggregates: interacting elements produce aggregate patterns to which those individuals in turn react to. This is precisely where the concept of emergence enters the picture. […]. While the mathematics required to solve the model [DSGE] may at times look tricky and intimidating, conceptually the model is disappointingly unrefined: starting from a discounted sum of infinite utilities and an intertemporal budget constraint, somewhere you will eventually find a marginal rate of substitution equating a relative price, and possibly an additional binding constraint that prevents the second-best from being achieved. Nothing is said about true heterogeneity in preferences and beliefs; the behavior of agents along disequilibrium paths; the net of non-market interactions linking agents; the insurgence of intratemporal and intertemporal coordination problems; in a nutshell, nothing is said about what really makes any macroeconomic system an object worth studying. (Delli Gatti et al., 2011, pp. 5-10)

Before moving on to the explanation of ABMs, it seems worthwhile to briefly discuss the most relevant logical inconsistencies of the mainstream approach pointed out by its critics.

3.2. Logical Inconsistencies of DSGE models

3.2.1. The SDM Theorem

The work of Sonnenschein (1972), Debreu (1974) and Mantel (1974) – henceforth *the SDM theorem* – can be summarized as follows:

Let the aggregate excess demand function $F(p)$ – obtained from the aggregation of individual excess demands $f(p)$ – be a mapping from the price index Π to the commodity space P^N.

Consider that a General Equilibrium is defined as a price vector p such that $F(p^*) = 0$.

Formally, the theorem states that the properties of the Walrasian aggregate excess-demand functions $F(\cdot)$ inherited from the individual excess-demand functions $f(\cdot)$, i.e., continuity, homogeneity of degree zero, Walras' Law (the total value of excess-demand is zero) and boundary condition (as prices approach zero, demand increases but does not go to infinity), are only sufficient to assure existence, but neither the uniqueness nor the local stability of p^*, unless preferences generating individual demand functions are restricted to very implausible cases (like the assumption that all agents in the economy have Cobb-Douglas preferences).

Thus, for a theory which claims to be rooted on general equilibrium, the mere fact that general conclusions could only be drawn for specific examples represents a reversal of ordinary logic (Delli Gatti et al., 2011).

3.2.2. The Representative Agent Hypothesis

A possible way out of the SDM theorem is founding the analysis on a fictitious representative agent. Actually, this assumption has been adopted massively by DSGE models. According to this approach, aggregate consumption is analyzed as if it were the consumption of a single individual, who is assumed to live forever, while the income and substitution effects of the whole economy are restricted to coincide with that of the representative agent. Similarly, labor market is usually treated as a single worker and the financial market as a single investor.

Thus, assumptions are made at the aggregate level, without saying anything about isolated individuals. In other words, the aggregate economy would behave like a rational individual, so that the macro-behavior simply becomes the sum of individuals-behavior and the aggregate properties can be detected at the micro-level as well. Even when the model allows for some heterogeneity, interactions that are not mediated by the price vector are generally ignored and coordination is ruled out by definition (Di Guilmi et al., 2017).

3.2.3. (In)Computability of General Equilibrium

To prove the existence of a General Equilibrium one must call the *Brower's fix point theorem*, i. e. by finding a continuous function $g(\cdot)\colon \Pi \rightarrow \Pi$ so that any fix point for $g(\cdot)$ is also an equilibrium price vector $F(p^*) = 0$.

Suppose we want to find an algorithm, which, starting from any arbitrary price vector p, chooses price sequences to check for p^* and halts when it finds it. In other words, to find the general equilibrium price vector $F(p^*) = 0$ means that halting configurations are decidable.

However, it violates the undecidability of the halting problem for Turing Machines and, therefore, the General Equilibrium solution is incomputable from a recursion theoretic view point.[34]

3.2.4. Price Mechanisms

In a General Equilibrium model all transactions occur at the same equilibrium price vector by construction. To achieve this outcome, economic theory has developed two mechanisms.

The first one, which is called the Walras' assumption, assumes that both buyers and sellers adjust (costless) their optimal supplies and demands to prices that are called out by a fictitious auctioneer, who keeps doing his job until a price vector which clears all markets is found. Only then transactions take place.

The second mechanism, which is called the Edgeworth's assumption, considers that buyers and sellers sign provisional contracts and are allowed to recontract (costless) until a price vector which makes all individual plans fully compatible is found. Once again, transactions will occur only after the equilibrium price vector is established.

No matter which mechanism one adopts, the general equilibrium model is one in which the formation of prices *precedes* the process of exchanges, instead of being the result of it. As Arrow (1959) pointed out, real markets work differently and in real time, so that the General Equilibrium could not be considered a scientific explanation of real economic phenomena.

3.2.5. Money

Introducing money into General Equilibrium models is at best problematic. Notice that, in this framework, a monetary trade should be the equilibrium outcome of market interactions among optimizing agents, so that no economic agent could decide to monetize alone. Moreover, the use of money implies that one individual gives up on

[34] Alan Turing proved, back in 1936, that it would be impossible to create a general algorithm to solve the halting problem for all possible program-input pairs. Briefly speaking, in computability theory, the *halting problem* determines, from an arbitrary computer program and an input, whether the algorithm will finish running or continue forever. A key feature of the proof was a mathematical definition of a computer and program: The *Turing Machine*. The halting problem is undecidable over Turing machines.

something valuable (e. g. his endowment or production) for something inherently useless (a fiduciary token for which he has no immediate use), expecting to advantageously trade it back it in the future.

However, remember that, in a General Equilibrium model, actual transactions occur only after a price vector that coordinates all trading plans has been found (costless). So, money can only be consistently introduced if the assumption of the absence of transaction costs is abandoned.

Following the same logic, credit makes sense only if agents can sign contracts in which one side promises future delivery of goods or services to the other side. Thus, in such models, markets for debts become meaningless; both information conditions and information processing requirements are not properly defined, and bankruptcy can be safely ignored (Delli Gatti et al., 2011).

3.3. Agent-Based Models

Are there other ways to perform macroeconomic analysis beyond that inspired by the DSGE approach? Many economists argue for a positive answer. In general, they understand that any economy – and, in particular, large economies composed of millions of individual entities – should be described as a complex, adaptive and dynamic system.

3.3.1. Introduction

In the Agent-Based framework, complexity takes place because of the dispersed and non-linear interactions between a large number of heterogeneous autonomous agents. In this view, aggregates cannot be deduced directly from an examination of the behavior of an individual in isolation. Thus, macro-behaviors may emerge from the market and non-market interactions, without them being a result of individual intentions.

According to Gallegati et al. (2017), ABMs allow the construction – based on simple evolving rules of behavior and interaction – of models with heterogeneous interacting agents, where the resulting aggregate dynamics and empirical regularities (i. e., emergent results) are not known *ex ante* and are not deducible from individual behavior.

The concept of economy as an evolving complex system is fundamental to this approach. Thus, it seems worthwhile to make some remarks. First of all, the term *evolving* means that the system is adaptive through learning. Agents' behavioral rules are not fixed.[35] Instead, they constantly change adapting to shifts in the economic environment in which they interact.

Notice that this is very different from its DSGE counterpart. As Gallegati et al. (2017, pp. 4-5) say:

> […] the traditional approach, which assumes optimizing agents with rational expectations, has been and is a powerful tool for deriving optimal behavioral rules that are valid when economic agents have perfect knowledge of their objective function, and it is common knowledge that all agents optimize an objective function, which is perfectly known unless there are exogenous stochastic disturbances. If agents are not able to optimize, or the common knowledge property is not satisfied, then the rules derived with the traditional approach lose their optimality and become simple rules. Moreover, they are fixed, that is, nonadaptive. In an ABM individual adaptive behavioral rules evolve according to their past performance: this provides a mechanism for an endogenous change of the environment. As a consequence, the "rational expectation hypothesis" loses significance. However, agents are still rational in the sense that they do what they can in order not to commit systematic errors. In this setting, there is still room for policy intervention outside the mainstream myth of optimal policies. Because emergent facts are transient phenomena, policy recommendations are less certain, and they should be institution and historically oriented.

The second explanation refers to the expression *complex system.* It means that the economic system has a high level of heterogeneity, that is, direct and indirect interactions that can generate *emergent properties* that are not inferred from the simple analysis of micro-relations.

This notion of *emergence* is what characterizes a complex system. In this approach, the dynamics of the *evolving agents* are aggregated regardless the need for special conditions for perfect aggregation and an always-in-equilibrium dynamic.

In fact, when dealing with complex economies, the key driver of evolution is not optimization but *selection*. In large interactive systems like this, individual decision processes become adaptive in the sense that agents can "change their mind" during the simulation process and often change the rule (Caiani et al., 2016; Delli Gatti et al., 2011).

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[35] However, it is not illegitimate to build ABMs with fixed rules whenever one wants to understand the dynamic of an economic system considering that agents behave in a certain way.

As shown in Schelling (1978), maximization behaviors may lead to lower payoffs than behaving reciprocally and cooperatively, whenever the enforcement of contracts is costly and exchanges happen through face-to-face bargaining.

The equilibrium of a system in an ABM does not require that all isolated elements be in equilibrium as well, but rather that the statistical distributions describing aggregate phenomena be stable. In other words,

> One of the objectives of an ABM simulation (but not the only one) is to make the joint distributions of economic agents converge in a suitable space of distributions. Even when fluctuations of agents occur around equilibrium, which we could calculate using the standard approach, the ABM analyses would not necessarily lead to the same conclusions. This is because the characteristics of the fluctuations would depend on higher moments of the joint distribution and often on the properties of the tails, or three kurtosis of the distribution. (Gallegati et al., 2017, p. 6)

This nonequilibrium dynamics prevents the complexity approach to deal with the common practice of *closing* the models through some exogenous imposition of a general equilibrium solution by means of some fixed-point theorems.

According to Delli Gatti et al., (2011, p. 16):

> The introduction of a Walrasian auctioneer inhibits the researcher from exploring the real question at stake in macroeconomics, that is, to explain how self-interested trading partners happen to coordinate themselves in decentralized markets most of the time, but also why from time to time some major economic disaster occurs without any apparent external cause. Complexity offers a way out of this situation, and it suggests new perspectives. Complex adaptive economies display a tendency to self-organize towards rather stable aggregate configurations, occasionally punctuated by bursts of rapid change. Spontaneous order emerges in the process of individual buying and selling transactions taking place in real space and time, without the need of any central controller. Adaptive and imitative behaviors give rise to stable and predictable aggregate configurations, as stability implies predictability and vice versa. Since it is sometimes safer to be wrong in the crowd than to be right alone, imbalances can now and then accumulate to the point that a bundle of chained bankruptcies becomes inevitable. After the bubble has burst and the system has experienced episodes of wild instability, new modes of adaptive behavior, technological opportunities and budget constraints co-evolve leading the economy towards a new phase of aggregate stability.

In traditional models it is assumed that one has a full characterization of individual preferences, that is, that one knows exactly what economic agents want. Payoff and utility function existence theorems are derived from this hypothesis, making it possible to represent the preference map of economic agents with scalar objective functions for which the maximum value exists.

In fact, according to Caiani et al. (2016), in traditional models the agents' optimization is a mere consequence of the initial hypothesis that the preference scheme of the agents is known *a priori*.

Tesfatsion (2006) summarizes it:

> Walrasian equilibrium in modern-day form is a precisely formulated set of conditions under which feasible allocations of goods and services can be price-supported in an economic system organized on the basis of decentralized markets with private ownership of productive resources. These conditions postulate the existence of a finite number of price-taking profit-maximizing firms who produce goods and services of known type and quality, a finite number of consumers with exogenously determined preferences who maximize their utility of consumption taking prices and dividend payments as given, and a Walrasian Auctioneer (or equivalent clearinghouse construct) that determines prices to ensure each market clears. Assuming consumer nonsatiation, the First Welfare Theorem guarantees that every Walrasian equilibrium allocation is Pareto efficient.
>
> The most salient structural characteristic of Walrasian equilibrium is its strong dependence on the Walrasian Auctioneer pricing mechanism, a coordination device that eliminates the possibility of strategic behavior. All agent interactions are passively mediated through payment systems; face-to-face personal interactions are not permitted. Prices and dividend payments constitute the only links among consumers and firms prior to actual trades. Since consumers take prices and dividend payments as given aspects of their decision problems, outside of their control, their decision problems reduce to simple optimization problems with no perceived dependence on the actions of other agents. A similar observation holds for the decision problems faced by the price-taking firms. The equilibrium values for the linking price and dividend variables are determined by market clearing conditions imposed through the Walrasian Auctioneer pricing mechanism; they are not determined by the actions of consumers, firms, or any other agency supposed to actually reside within the economy. (Tesfatsion, 2006, pp. 175-176)

On the other hand, agent-based approach:

> […] aims to study economic phenomena in their complexity; taking into account joint distributions of individual characteristics, the direct and not only indirect interactions and therefore the way in which economic networks are made and changed. Hence, it cannot assume to be able to perfectly represent the preferences and therefore to have an exact knowledge of the objective functions. The starting point of each agent-based analysis is a description of the rules of behavior, that is, the map between the actions and the information set available to them in which a partial knowledge of the objective functions is included. These rules can be derived from empirical work, from economic experiments, from studies carried out in disciplines other than economics (psychology, sociology, etc.) or from a purely normative analysis." (Caiani et al., 2016, p. xiv)

Furthermore, as Gallegati et al. (2017) argue, the evolutionary process of differentiation, selection, and amplification loads the system with novelty and is responsible for its growth in order and complexity. The mainstream approach has no such a mechanism to endogenously create novelty or generate growth in order and complexity. In their words:

> […] results of the ABM model are new because they take into consideration a very important element of economic systems: the networks of direct and indirect interactions, which are often extremely complex and not approximated by simple graphs such as random graphs. Real economies are composed by millions of interacting agents, whose distribution is far from being a simple transformation of the "normal" one. (Gallegati et al., 2017, p. 7)

Summarizing, the Agent-Based approach may offer new paths to new and old unsolved questions. Of course, it is still in a far too premature stage to offer definitive tools, although it has already yielded interesting results, especially when analyzing complex situations that are difficult to investigate with DSGE models.

However, addressing the complex view to macroeconomics requires appropriate conceptual and analytical tools. According to Delli Gatti et al. (2011, p. 17):

> The abandonment of the Walrasian auctioneer implies that market outcomes must be derived from the parallel computations made by a large number of interacting, heterogeneous, adaptive individuals, instead of being deduced as a fixed-point solution to a system of differential equations. The process of removal of externally imposed coordination devices induces a shift from a top-down perspective towards a bottom-up approach.

In this bottom-up approach, individual behavior is modeled according to simple behavioral rules and agents are allowed to have *local interaction* and to change the *individual rule* (through adaptation) as well as the *interaction nodes*. Aggregation allows the emergence of some *statistical regularity*, which cannot be inferred from individual behavior (*self-emerging regularities*). This *emergent behavior* feeds back to the individual level (downward causation) thus establishing a macrofoundation of micro (Colander, 1996). The Agent-Based approach aims precisely to describe, in a reduced scale, the behavior of single individuals and bring out the aggregate properties (Caiani et al., 2016).

Consequently, each and every proposition can be falsified at micro, meso and macro levels. The distance between the Agent-Based approach and the DSGE one is large. According to Caiani et al. (2016, Introduction, p. xvi):

> In an ABM, the interactions are governed by rules of behavior that the modeller codifies directly in the individuals who populate the environment. In an ABM, the behavior is the point in which a modeller begins to make hypothesis. The DSGE modelers make assumptions about what an optimizing agent wants, compatibly with budget and resource constraints, and represent these wishes with concave real-valued functions defined over convex sets. Based on the combination of objectives and constraints, the behavior is derived by solving the first-order conditions and when necessary also the second-order conditions. The reason why economists set their theories in this way - making assumptions about the goals and then drawing conclusions about the behavior - is that they assume that the allocations, decisions and choices are guided by individual interests. Decisions and actions are carried out with the aim of reaching a max-min goal. For consumers, this usually regards utility maximization; a purely subjective assessment of well-being. For businesses, the goal is typically to maximize profits. This is exactly where rationality, for DSGE, is manifested in economics. In a nutshell, in DSGE models the modeller sets the objective function and the consequent maximization generates the rules. In the ABM the modeller sets directly the behavioral rules given empirical evidence and experiments that should control the degrees of freedom.

After our effort to systematically introduce the Agent-Based approach, we shall now move forward in our way to understand ABMs by exploring their main features, so that we can start to work on their common structure, as well as the issues of empirical validation and estimation of ABMs.

3.3.2. Main Features of ABMs

The basic units of ABMs are the "agents". In a nutshell, they are called agents because they are autonomous objects[36] which interact with each other and with the environment. In economics, agents can involve anything from individuals to more sophisticated units, such as social groups (e. g., families and firms). Agents can also be more complicated organizations, like banks, industries or even countries. Lastly, agents can be composed by other agents, as long as they are perceived as a unit from the outside and actually do something, having the ability to act and possibly react to external impulses and interact with the environment and with other agents.

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[36] By autonomous objects we mean that no central control is required to ensure the system's dynamics, that is, there is no *top-down* control over individual's behavior.

Regarding the environment, it may include physical entities (like infrastructures, geographical locations, etc.) and institutions (like markets, regulatory systems, etc.). It can also be modeled as if it were an agent (e. g., a central bank), whenever the above conditions are satisfied. If not,

> […] it should be thought of simply as a set of variables (say "weather" or "business confidence") characterizing the system as a whole or one of its parts. These variables may be common knowledge among the agents or communicated throughout the system by some specific agent – say the statistical office – at specific times. […] it should be clear that aggregate variables like consumption, savings, investment and disposable income, which are the prime units of analysis of Keynesian macroeconomics, cannot be modeled as agents in an Agent-Based framework as they are computed by aggregating microeconomic agent quantities; the same applies to the fictitious representation of a representative agent, a cornerstone of neoclassical economics. […]. The direct modelling of a demand or a supply curve is also forbidden in an agent-based setting: rather, these aggregate functions may (or may not) emerge as the outcome of the decisions of the individual agents. (Richiardi, 2018a, p. 11)

In short, according to the Agent-Based literature, we can summarize the ABM fundamental characteristics into three main tenets. The first is that there is a multitude of objects that interact with one another and also with the environment. The second is that the objects are autonomous, as previously explained. The third is that the outcome of their interaction is numerically computed.

In fact, ABMs may have other characterizing features. According to Epstein (2006), we can list the following key-elements: *heterogeneity*, *explicit space, local interactions, bounded rationality, and nonequilibrium dynamics*. Let us briefly explain each one.

3.3.2.1. Heterogeneity

Agents are explicitly modeled and can differ one from another. While in analytical models the reduction in the ways individuals can differ can be a big advantage, in ABMs – due to computational developments – it is possible to specify different values of the parameters (e. g. preferences, endowments, location, social contacts, abilities, etc.) for different individuals. Normally, this is done by choosing a suitable distribution for each relevant parameter, so that a limited number of parameters (those generating the distribution) are added to the model.

3.3.2.2. Explicit Space

This can be seen as a specification of the previous point. The space in which units act and interact is explicitly modeled, i. e., individuals often differ in the physical place where they are located, and/or in the neighbors with whom they are allowed to interact. It defines the network structure of the model and the concepts of *local* and *neighborhood.*

3.3.2.3. Local Interactions

Again, this can be seen as a specification of the network structure linking the agents. The actions of the agents are local since they do not interact with the totality of the system, but only with their neighbors, which may be referred to their spatial, economic, or social position. Analytical models often assume either global interaction (as in Walrasian markets), or very simple local (e. g., 2x2) interaction arrangements. ABMs allow for much richer interaction specifications.

3.3.2.4. Bounded Rationality

In models based on general equilibrium solutions, it is usually easier to implement some form of optimal behavior rather than solving models where individuals base their decisions on *reasonable rules of thumb* or learn from the experience of others. In ABMs, *bounded rationality*[37] enters the scene because agents base their decisions on simple heuristics based on local information, since they are not endowed either with perfect information of the functioning of the system where they live or with infinite computing capacity to process all the available information. [38]

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[37] According to Russo et al. (2018, p. 108), "bounded rationality can be considered an alternative behavioral paradigm for economic agents, as opposed to the neoclassical hypothesis of constrained maximization. Indeed, in a complex environment in which information is limited and incomplete, the behavior of agents tends to be based on heuristics, that is, relatively simple rules of decisions that agents use to try to reach a satisfying choice. Moreover, agents may learn from their behavior and from the interaction with other agents and the environment. When we consider the economy as a whole, we must consider that agents can directly interact with other agents when taking decisions or learning about the working of the economy. The interaction of heterogeneous agents can lead to complex dynamics at the level of the whole system. For this reason, the macro level can be different from the simple sum of micro entities."

[38] Even in cases where limited information is taken into account, the mainstream approach faces big challenges. See Delli Gatti et al. (2011, p. 21): "According to the mainstream approach, information is complete and free for all the agents. Note that one of the key assumptions in the Walrasian tradition is that

3.3.2.5. Nonequilibrium Dynamics

General equilibrium models assume continuous market clearing, so that every out-of-equilibrium dynamics is discarded from the beginning, and initial conditions do not matter. On the other hand, in ABMs, equilibrium is not treated as a natural state of the system. From an analytical viewpoint, the latter are recursive (stochastic) dynamic models[39], in which the state of the system at time $t + 1$ is computed (or probabilistically evaluated) starting from the state at time t. Nonequilibrium dynamics are of central concern. In many cases, model dynamics are not ergodic because the initial conditions and path dependency matter. Thus, they allow the investigation of what happens all along the route, not only at the start and at the end of the journey. Therefore, they allow system analysis in which equilibrium may not even exist.

3.3.3. The Structure of a Basic ABM

In what follows, we intend to provide a basic manageable theoretical formalism for ABMs, based on the works of Delli Gatti et al. (2011) and Richiardi (2018a; 2018b).

Briefly speaking, ABMs almost always follow a standard approach. First, they are initialized with parameters which define the starting situation. Then, the model is executed to simulate the passage of time. Each step represents a shot time duration (e. g. a day) at which each agent performs some action (or simply does nothing), according to his behavioral rules. Finally, interaction takes place and may include a lot of things, such as communicating with other agents, changing or moving through the environment, etc.

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any strategic behavior is ruled out, and the collection of the whole set of the information is left to the market via the auctioneer. In fact, one could read the rational expectation "revolution" as an attempt at decentralizing the price setting procedure by defenestrating the auctioneer. Limited information is now taken into account, but the constraints have to affect every agent in the same way (the so-called Lucas' islands hypothesis), while the Greenwald-Stiglitz theorem (Greenwald and Stiglitz [1986]) states that in this case the equilibrium is not even Pareto-constrained. If information is asymmetric or private, agents have to be heterogeneous and direct interaction has to be considered: this simple fact destroys the efficiency property of mainstream model and generates coordination failures. On the contrary, ABMs are built upon the hypothesis that agents have limited information and learn through experience and by interacting with other agents."

[39] A discussion about recursive systems is beyond the scope of this work. For our purposes, it is enough to say that a recursive system is one in which the output is somehow dependent on one or more of its past outputs. See Delli Gatti et al. (2018, pp. 33-42) for a detailed explanation.

The program keeps running, step by step, until either some programmed halting condition is met or the modeler stops the simulation manually. While the program is in execution, the outcomes of agents' interactions can be measured on graphs or monitors.[40]

3.3.3.1. Setting the Stage

In order to turn the complexity of the real world into manageable theoretical frameworks, scientists usually abstract some of the characteristics of the particular elements they observe and group them into "classes".

Assumption 1: The starting point of our analysis is a methodological assumption according to which any economic system consists of "classes". Each class contain a very large number $\boldsymbol{N}$ of agents (objects) who are heterogeneous according to a certain number $\boldsymbol{n}$ of different criteria (attributes).[41]

An example makes it clearer. Consider an economy in which there are three classes: firms, households and banks. Firms are units that use their productive inputs to produce final goods, choosing quantities and prices. Objects that belong to the class "firms" are characterized by $n = 3$ different attributes: heterogeneity of size, financial condition, and technology. Objects of the class "households" (units which offer labor, consume goods and save precautionally) are characterized by $n = 3$ attributes: different employment status, labor income, and savings. Finally, objects belonging to the class of banks (units which provide funds) are characterized by $n = 1$ attribute: different internal financial conditions.

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[40] Certainly, one can simply use an ABM that has already been developed. However, it is much more interesting to understand the program code that is making the model works. Fortunately, advances in programming languages and more friendly interfaces have made programming accessible to those without expert knowledge. See Hamill and Gilbert (2016) for a very elucidative tutorial on how to build a simple ABM using a software called NetLogo.

[41] According to Delli Gatti et al. (2011, p. 30), "in formal terms, an object is an algorithmic description (in our case, lines of software code inside a larger computer program) of a purposive entity with some identifiable and specialized features. Each object contains a list of attributes and a set of methods acting on these attributes. An object can control the mode of external access to its attributes and methods, by declaring them public (accessible to all), private (inaccessible to all) or protected (accessible only to some other objects). A class is then defined as a template or blueprint for the instantiation of objects sharing those common but peculiar features."

Summarizing, the first stage is to classify agents in general types, allowing for *substantial heterogeneity* of individual characteristics. [42]

However, models can easily become unmanageable as heterogeneity takes place. For instance, imagine a situation where both the criteria and the types asymptotically tend to infinity (i. e., $n \to \infty$ and $N \to \infty$). Clearly, pushing heterogeneity of objects and attributes to these limits is neither useful nor realistic. Dealing with the issue of finiteness is therefore fundamental for the usefulness of models.

In fact, there is no compelling reason to propose and follow a particular rule in picking up finite values of n and N inside the infinite set of integers. However, the preference is models with a relative small n (but $n \gg 1$) and a relative large N, that is, models with many types of agents but a relatively small set of criteria to classify agents by type. This choice is dictated by the preference for realism in a relatively simple and manageable setting.

But how small should n be? This is actually a matter of convenience that depends on what the modeler wants to analyze. For instance, suppose that the core of the analysis is the long-run growth. Thus, heterogeneity in technology adoption becomes essential.

And how large should N be? Preferentially the largest possible, subject to the constraints of computational power. However, sometimes it is not needed to specify a particular class of agents in a very detailed way. There are situations where a binary choice may be enough, by recurring to the presence or absence of some qualitative features (e. g., employed/unemployed).

3.3.3.2. Rules of Behavior

Once classes and agents are created, the modeler needs to specify the agent's rule of behavior, that it, how agents are allowed to process information, and to act consequently.

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[42] This is not exactly a novelty of macroeconomic models. In fact, in the Overlapping Generations (OLG) framework, agents differ because of their age. They can either be young or old. This is a simple example where there is $n = 1$ criterion of classification and there are $N = 2$ types of agents. According to Delli Gatti et al. (2011, p. 30), "of course there can be many agents in any generation, but they are ultimately uniform. Each young (old) is a clone of any other young (old). Therefore, despite the appearance of a very large number of agent, the model boils down to only two of them".

Assumption 2: Agents are characterized by simple *behavioral rules* (methods acting on attributes), that is, stylized (algorithmic) patterns of economic behavior. Each agent may follow different – say, ν – rules due to different circumstances, i.e. different time periods, geographical areas, markets, and so on.

These rules may or may not be the outcome of an optimizing process. But notice that optimization yields the smallest possible set of rules. The optimal behavior will be the only one behavior the agent can rationally follow because of the fully specification that is required for any optimization procedure (i. e., objective functions, constraints, and information sets). In symbols, that situation yields $\nu = 1$.

However, once we choose to get rid of optimization, we can easily get lost in the wilderness of behavioral rules, as there are no constraints on the number of behavioral rules one can use. The borderline situation, where $\nu \rightarrow \infty$, would push heterogeneity to the limit where the model would become easily unmanageable.

In fact, agent-based modelers prefer models with a relatively *small* ν. In other words, the preference is for models with many types of agents but a relatively small set of behaviors for each class of agents. This preference is based on realism and manageability. Indeed, it is very unlikely that agents are so sophisticated to adopt a large number of different behavioral rules. It is much more reasonable to assume a relatively small ν.

The choice of the behavioral rule can be done according to the empirical literature, or – when possible – by simply asking people (survey studies). When there is no clue coming from the empirical literature, the common choice is to run the model with several behavioral rules and to compare them in terms of how good they are in generating results fitable into the available empirical evidences. According to Delli Gatti et al. (2011), the rule that comes first in its ability to reproduce stylized facts is the one to be adopted.

At a very broad level, as Catullo et al. (2016) argues, what is important for the choice of an economic agent is his/her information set or state at time t.

Assume that the information set at time t is given by:

$$\Omega_{i,t} = (x_{i,t}, e_{i,t}, n_{i,t})$$

where $x_{i,t}$ is a vector that collects individual characteristics (e. g. preferences, technology, etc.); $e_{i,t}$ is a vector of external signs (e. g. knowledge of the market in which they operate, etc.); and $n_{i,t}$ is the neighborhood of agent i that one observes. All the variable are predetermined, i. e., observable variables at time t of the agent's choice.

Define a rule as an *action/choice given the state/information set at time* t. Also, consider the fact that agents behave rationally in the sense that they learn from their mistakes (i. e., rules have a feedback structure).

As in ABMs the environment is complex, becoming it difficult to find an optimal rule, the use of micro and experimental evidence with some reinforcement mechanism is needed. This notion of rationality is better specified by using the definition of *constructive rationality*, as in Tesfatsion (2016). According to this definition, there is constructive rationality when the action/rule of an agent can be expressed as a function of the state at time t, that is, when in the state there are consideration of future events, those have to enter as anticipations (i. e., function of the state at time t).

A behavioral rule is then a relationship between an action (i.e., a specific level of a *control* or *decision variable*) and the levels of the *state variables* that characterize the agent. Let us make it clearer by presenting some formalization.

Suppose, for simplicity, that there are $\boldsymbol{k}$ control variables and $\boldsymbol{\sigma}$ state variable for each agent. Let underscores denote vectors, so that $\underline{\boldsymbol{C}}_{\boldsymbol{i,t}}$ is the $(\boldsymbol{k}, \mathbf{1})$ vector of control variables available to agent $\boldsymbol{i}$ in period t and $\underline{\boldsymbol{S}}_{\boldsymbol{i,t}}$ is the $(\boldsymbol{\sigma}, \mathbf{1})$ vector of state variables which characterize the agent in the same period.

Notice that since the current action is likely to affect the state in the same period, logically, it will be affected by the state of the agent in the past. Hence, the simplest conceivable formulation for a behavioral rule is:

$$\underline{C}_{i,t} = C_i(\underline{S}_{i,t-1}) \tag{112}$$

However, the action in period t will contribute to the current state of the agent and therefore to the future action that the agent will take. Hence,

$$\underline{S}_{i,t} = F_i\left(\underline{S}_{i,t-1}, C_{i,t}\right) = F_i\left(\underline{S}_{i,t-1}, C_i\left(\underline{S}_{i,t-1}\right)\right) \tag{113}$$

The equation above represents the *general law of motion* of the state variables which rule the evolution over time of the features characterizing the individual agent. It is a σ-dimensional generally non-linear dynamical system, which maps the overall state of the agent in t ($\underline{S}_{i,t}$). Indeed, there are N different dynamical laws, one for each agent. Therefore, the evolution over time of the macroeconomy is described by a system of $N \times \sigma$ difference equations.

By construction, this system is so far composed by unrelated equations, since each individual's behavioral is considered as isolated from the others. In other words, each agent is evolving in isolation. Of course, it is unrealistic to conceive an economy like that. So, we have to move one step further in the direction of modelling economic behavior in a fully integrated macroeconomic system by taking into account the fundamental role of interaction.

3.3.3.3. Interaction

In order to introduce the issue of *social interactions* in the simplest way, we shall assume the following:

Assumption 3: The actions of the agent $\underline{C}_{i,t}$ in t are affected by the collective actions of other agents in the past.

To capture it, we will use the vector of some summary statistics (e. g. the average) of cross-sectional control variables, $\underline{E}_{t-1}$. Following Delli Gatti et al. (2011), we will assume that individual control variables are not affected by other agents' *individual* actions in a strategic framework. It means that, on the one hand, the i^{th} agent is indeed affected by the population consisting of the $N-1$ remaining agents (with N large enough, so that the contribution of agent i to the aggregate becomes *negligible*) and he/she may be also aware of this influence, but he/she is not intentionally "playing" a game against each of the other individuals. On the other hand, agent i is surely contributing to shape the state of the j^{th} agent, but only as one tiny component of an aggregate that is meant to describe the colletive behavior of the population at large.

With this assumption, reformulating (112) gives us:

$$\underline{C}_{i,t} = C_i(\underline{S}_{i,t-1}, \underline{E}_{t-1}) \tag{114}$$

Therefore, equation (113) must be "augmented" as follows:

$$\underline{S}_{i,t} = F_i(\underline{S}_{i,t-1}, C_{i,t}) = F_i(\underline{S}_{i,t-1}, C_i(\underline{S}_{i,t-1}, \underline{E}_{t-1})) \tag{115}$$

Notice that the presence of the vector $\underline{E}_{t-1}$ in the expression above captures the idea that the state of the individual is affected by an average of all the actions taken by all other agents.

Equation (115) represents the law of motion of the state variables which govern the evolution of the characteristics of each individual agent. Notice that, by construction, now the system is not strictly individual. In other words, each agent's state is evolving over time considering not only his own past states, but also the average state of the economy, represented by $\underline{E}_{t-1}$.

3.3.4. Obtaining Results in ABMs

With the above explanations and following the formalism of Richiardi (2018b), we will offer a formal characterization of ABMs and analyze how they can give us satisfactory results.

First, assume that, at each time t, an agent $i : i \in 1, \ldots, n$, is fully described by some state variables $\sigma_{i,t} \in \Re^k$. Let his/her state variables evolve by the following difference equation:

$$\sigma_{i,t+1} = f_i(\sigma_{i,t}, \sigma_{-i,t}, \theta_i, \xi_{i,t}) \tag{116}$$

where $\xi_{i,t}$ are stochastic terms, and $\theta_i \in \Theta$ is a vector of parameters, with Θ being a compact subset of $\Re^Q$.

The behavioral rules may be individual-specific both in the functional form $f_i(\cdot)$ and in the parameters θ_i, and may also depend on the state σ_{-i} of all agents other than i. The set of structural equations (116), defined at the individual level, defines the data-generating process (DGP) of the model.

At any point in time, the system is in a state $\mathrm{X}_t = (\sigma_{i,t})$ which is the matrix of all individuals states. By replacing (116) in the definition above, we obtain:

$$X_{t+1} = F(X_t, \theta, \Xi_t) \tag{117}$$

where Ξ_t is a matrix which contains all stochastic elements at time t.

Equation (117) defines the *transition equation* of the system. Note that, in optimal control, there is a distinction between *state* and *control* variables, i. e., the latter are subject to the optimizer's choice and have influence on the value of the state variable of interest. Contrariwise, in ABMs there is no need to distinguish between them, as agents do not really engage in mathematical optimization. So, we can simply refer to both state and control variables as *state variables* $x_i = [\sigma_i, k_i]$. Each individual variable evolves according to a certain rule, or law of motion f_i.

In order to investigate some aggregate (observable) statistics of the economy, we shall define a vector of aggregate variables y_t as a function over the state of the system, that is, a projection from X to y:

$$y_t = m(X_t, e_t) \tag{118}$$

where e_t represents extra random terms that account for measurement errors and other shocks to the observables, if any.

Equation (118) is the *measurement equation*. The *state-space representation* of the system is formed by equations (118) and (117). To solve equation (118) for each t, regardless of the specification of $f_i(\cdot)$, we need to use *backward iteration*, i. e., to trace the stochastic evolution y_t back to the initial state of the system and the values of the parameters.

Expliciting this relationship is difficult because of the random terms Ξ and e that enter at every stage. As the behavioral rules f_i and the measurement function (118) do not need to be linear, these random terms cannot be netted out by taking expectations.

The only way to investigate the mapping of (X_0, θ) into y_t is by means of Monte Carlo analysis, i. e., by simulating the model for different initial states and values of the parameters, and repeating each simulation over and over again until the obtainment of the distribution of y_t.

To go further into the explanation, we can think about how the model simulation works on a digital computer. Because digital computers are deterministic machines, random terms are not truly random, that is, they are generated by an algorithm which produces sequence of numbers that resemble the properties of random numbers. This sequence is called *pseudo-random* and the algorithm is referred to as *random number generation*. Each sequence is identified by a seed, which is often called *random seed*. Specifying the random seed guarantees the reproducibility of the results.

Therefore, the random terms Ξ and e are a deterministic function of the random seed s, and equations (117) and (118) become:

$$X_{t+1} = F(X_t, \theta, s) \tag{119}$$

$$y_t = m(X_t, s) \tag{120}$$

The random seed can be thought of as a further initial condition: $Z_0 = (X_0, s)$. By iteratively substituting X_{t+1} with X_t using equation (119), we get:

$$\begin{gathered} X_t = F(F(\dots F(Z_0, \theta) \dots)) \\ X_t = F^t(Z_0, \theta) \end{gathered} \tag{121}$$

$$\begin{gathered} y_t = m(F^t(Z_0, \theta)) \\ y_t = g_t(Z_0, \theta) \end{gathered} \tag{122}$$

Notice that the law of motion (122) uniquely relates the value of y - at any time t - to the initial conditions of the system Z_0 and to the values of the parameters θ. Equation (122) is known as the *input-output transformation (IOT) function.*[43] This equation is the basic object of interest to yield results in an agent-based approach.

Moreover, notice that (122) completely satisfied, so that it is possible to explore their local behavior by analyzing the artificial time series produced by the simulation (Richiardi, 2018b). Because the IOT function has no analytical formulation, it has to be analyzed by computer simulations.

3.3.5. Empirical Validation of ABMs

Broadly speaking, the validation process involves a judgement over the quality of the model. However, it is not a trivial process. First of all, it is important to say that no model exists without an underlying theory. So, a model can be good (adequate) from a point of view, and bad (inadequate) from another one. Moreover, model validation can be defined along two dimensions.

A first dimension is the validation of the model relative to the theory, that is, whether the model is consistent with the theory on which it relies on. This is called *concept validation*. When applied to computational models, the concept validation requires an additional level of validation – the *program validation* – i. e., the validation of the code that simulates the model (relative to the model itself).

The second dimension is the validation of the model against real data. This procedure is called *empirical validation.* The aim of this section is to introduce the main techniques of empirical validation of ABMs in economics.

In a nutshell, empirical validation may concern the model inputs and/or outputs. The *input validation* refers to the realism of the assumptions, whilst the *output validation* investigate the plausibility of the data generated by the model, that is, whether the model delivers output data that resembles, somehow, real-world observations.

As we shall notice, input and output validations are connected, as the latter represents a joint test on the structure of the model and the values of parameters. As a

[43] The word "function" is appropriated here. Note that for any given input, the computer model will give only one output (though, different inputs might lead to the same output).

matter of fact, output validation can be applied to refine the parameters of the model. This procedure is called *calibration* or *estimation*[44] (Fagiolo and Richiardi, 2018).

3.3.5.1. Input Validation of ABMs

Validating the inputs of an ABM consists of checking whether the building blocks of the model and its assumptions are in line with the available evidence. Although input validation in an important step in model building, so far no explicit technique has been proposed to perform such a task in a formal and consistent way (Fagiolo and Richiardi, 2018).

In fact, any practice that intends to ensure that the fundamental conditions added into the model resemble aspects of the real life is doomed to face the issue of input validation. Here, we will consider input validation as a practice to ensure that the fundamental, structural, behavioral, and institutional conditions (i. e., assumptions about the rules of behavior and interaction) incorporated into the model are in tune with what is observed in the reality. According to Delli Gatti et al. (2011), input validation can be interpreted as an *ex-ante validation* (i. e., the researcher tries to introduce the correct parameters in the model before actually running it).

The information for input validation can be gathered from lab experiments, case studies, and actual empirical data collected at the micro level, for instance.[45]

3.3.5.2. Output Validation of ABMs

Indeed, the debate on ABMs validation is still very open and a new wave of approaches to empirical validation has recently flowered in the agent-based literature (*Fagiolo* et al., 2017). According to Windrum et al. (2007), the issue of empirical validation of AB models depends two factors: a) ABMs invariably contain non-linearities, stochastic dynamics, non-trivial interaction structures among economic agents, and micro-macro feedback which open up a whole set of methodological problems associated

[44] Although we are using calibration and estimation as synonyms, there are differences in the two practices, as we will show in section 3.3.6.
[45] See Hommes and Lux (2013) for the use of lab experiments. Also, see Cirillo et al. (2007) for the use of empirical data.

to the relations between the "real-world data generating process" (rwDGP) and the "model data generating process" (mDGP)[46]; b) heterogeneity in empirical validation procedures might also be due to the lack of standard techniques for crafting and analyzing ABMs. The reasons often pointed out for the lack of standard protocols are partly due to the high level of heterogeneity characterizing the process of constructing and analyzing ABMs. Moreover, there is no consensus about how - and if - ABMs should be validated[47].

In fact, a number of ABMs mostly engage in purely qualitative theorizing and are not empirically validated in any meaningful sense (Fagiolo and Richiardi, 2018). In other words, most ABM efforts do not go beyond a *proof of* concept, with no rigorously tests using empirical data (Janssen and Ostrom, 2006). The focus of such models is the analysis of qualitative aggregate patterns (e. g. the emergence of coordination and cooperation). Although forecast exercises are possible, they usually yield unpredictability. Thus, these models are not frequently "taken to the data".[48]

When the model is suited for empirical validation, output validation becomes a tool for the comparison of instances of the model with different parameter values and a choice of the one that better "fits the data". Agent-based modelers almost never intend to estimate or calibrate a model using a unique optimal choice for all the parameters. Rather, they look for confidence intervals or ranges of the relevant parameters. When the goal is more descriptive, they rather aim at identifying a reasonable and relatively small subset of the parameter space where counterfactual types of questions can be asked (Fagiolo and Richiardi, 2018).

The calibration/estimation of ABMs tries to deal with the problem of overparametrization by reducing the space of possible sets to be explored. This is done by using empirical data. Indeed, there are different approaches used in the agent-based literature to deal with this issue, but they all attempt to restrict the parameters so that the model outputs resemble the real output of interest as closely as possible.

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[46] It is required that the mDGP be simpler than the rwDGP and – in simulation models – the mDGP must generates a set of simulated outputs. The extent to which the mDGP is a good representation of the rwDGP is evaluated by comparing the simulated outputs of the mDGP with the real-world observations of the rwDGP.

[47] Some researchers engaged in qualitative modeling are critical of the suggestion that meaningful empirical validation is possible at all (see Valente, 2014).

[48] However, sometimes appropriate extensions/modifications of qualitative models can be empirically tested.

There are approaches that are mostly qualitative and those which rely on quantitative methods to estimate/calibrate the parameters. The latter try to identify the most-likely parameters ranges based on observed qualitative similarities between the real world and the model outcomes.

In the next section we will briefly review some of the most-used qualitative calibration techniques, whereas in the last section we will deal with some of the most promising quantitative approaches.

3.3.5.2.1. Qualitative Output Validation Techniques

3.3.5.2.1.1. The Indirect Calibration Approach

Drawing upon a combination of stylized facts and empirical datasets, the Indirect Calibration is a pragmatic four-step approach to empirical validation. As its name suggests, one must first perform validation, and then indirectly calibrates the model by focusing on the parametrization that are consistent with output validation. In what follows, we give a briefly description of the four steps.

In the first step, the modeler identifies a set of stylized facts[49] that concerns his interest in reproducing and/or explaining with a model. In the second step - along with the prescriptions of the empirical calibration procedure – the researcher builds the model in a way that the microeconomic description keeps as close as possible to empirical and experimental evidences concerning microeconomic behavior and interactions. This step implies collecting all possible evidences about the underlying principles that apprise real-world behaviors so that the microeconomic level is modeled in a "not-too-realistic fashion" (Windrum et al., 2007).

The third step involves restricting the space of parameters by using the empirical evidence on stylized facts (and also restricting the initial conditions, if the model turns

[49] Stylized facts typically concern the macro-level (e.g., the relationship between unemployment and GDP growth) but can also be related with cross-sectional regularities (e.g., the shape of the distributions on firm size) (Windrum et al., 2007). According to Fagiolo and Richiardi (2018, p. 173), "by emphasizing the reproduction (explanation) of a set of stylized facts, one hopes to circumvent problems of data availability and reliability. However, in order for empirical validation to be effective, the stylized facts of interest should not be too stylized, or too general. Otherwise, they might not necessarily represent a difficult test for the model: the model might pass the validation procedure without providing any effective explanation of the phenomena of interest."

out to be non-ergodic). This step is the most sensible as it implies a fine sampling of the parameter space. In fact, it is an exercise in "indirect calibration". It is also computationally demanding and requires the use of Monte Carlo techniques.

The fourth and last step is where the researcher should deepen his understanding of the causal mechanisms that underlie the stylized facts under scrutiny and/or explore the emergence of new stylized facts – i.e. statistical regularities that are different to the stylized facts of interest – which the model can validate *ex post*. This procedure might be done by further investigation of the subspace of parameters that resist to the third step, that is, those consistent with the stylized facts of interest.

The underlying goal of the indirect calibration approach is to investigate whether the model is able to reproduce jointly a wide range of macroeconomic and microeconomic stylized facts. If the model successfully matches empirical regularities concerning industrial dynamics as well as more structural relations between macroeconomic aggregates, this ought to be taken as a robust empirical validation (Fagiolo and Windrum, 2007; Fagiolo and Roventini, 2012, 2017; Fagiolo et al., 2017), offering plausibility to its use as a "computational laboratory" to test different policy experiments.

3.3.5.2.1.2. The History-Friendly Approach

This approach offers an alternative to the problem of overparametrization, by bringing modelling more closely in line with the empirical evidence. Notice that it is very similar to the indirect calibration discussed above. The key difference is that this approach uses the specific case study of an industry to model parameters, agent's interactions, and agent's decision rules. In other words, it is a calibration approach that uses particular historical features as a type of model calibration.

The working process of this approach is perfectly described in Windrum et al. (2007, p. 22):

> Through the construction of industry-based AB models, detailed empirical data on an industry inform the agent-based researcher in model building, analysis and validation. Models are to be built upon a range of available data, from detailed empirical studies to anecdotal evidence to histories written about the industry under study. This range of data are used to assist model building and validation. It should guide the specification of agents (their behavior, decision rules, and interactions), and the environment in which they operate. The data should also assist the identification of initial conditions and parameters on key

> variables likely to generate the observed history. Finally, the data are to be used to empirically validate the model by comparing its output (the `simulated trace history') with the `actual' history of the industry. It is the latter that truly distinguishes the history-friendly approach from other approaches. Previous researchers have used historical case studies to guide the specification of agents and environment, and to identify possible key parameters. The authors of the history-friendly approach suggest that, through a process of backward induction one can arrive at the correct set of structural assumptions, parameter settings, and initial conditions. Having identified the correct set of 'history-replicating parameters', one can carry on and conduct sensitivity analysis to establish whether (in the authors' words) 'history divergent' results are possible.

Albeit enthusiastic, it is important to say that both the indirect calibration and the history-friendly approaches raise a set of fundamental methodological issues. However, it is beyond the scope of this work a detailed methodological analysis of empirical validation.[50]

3.3.6. Estimation of ABMs

ABMs are in general complex non-linear models, and can thus present many different behaviors depending on the region of the parameter space being sampled. Therefore, accessing the performances of the model in the right region of the parameter space is crucial for model evaluation. Once this task is done and the model is considered appropriate for its scopes, lessons may be learned about what might happen in the real world if some of the parameters is changed, either as a consequence of some unforeseen developments (*scenario analysis*) or due to some specific action purposefully implemented (*policy analysis*).

The objective, broadly speaking, is the comparison of instances of the model with different parameters values in order to select those which better fit the data. Before the discussion of some estimation methods used in the agent-based approach, it is worthwhile to make two remarks.

First, although many researchers argue that calibration and estimation are basically the same thing, others believe that calibration is something different from estimation. As Richiardi (2018c) argues, the differentiation boils down to a matter of convenience. For our purposes, it is helpful to distinguish them along the following lines:

[50] For a detailed analysis of methodological issues regarding empirical validation of ABMs, see Windrum et al. (2007). See also Fagiolo and Richiardi (2018).

> […] *calibration* aims at maximizing the fitness of the model with the observed data in a distance metric arbitrarily chosen by de modeller, without bothering about the 'true' value of the parameters of the real world data generating process (rwDGP), or the uncertainty surrounding them; *estimation* aims at learning about the 'true' value of the parameters of the rwDGP by evaluating the fitness of the model with the observed data in a carefully chosen distance metric, such that the estimator has well known (at least asymptotically) properties. Roughly speaking, maximization of the fitness is a goal in calibration, a mean in estimation. Calibration is meant to show that the model is *plausible* – that is, it resembles the real world – and aims at reducing the number of possible worlds, one for each combination of the parameters, that have to be explored in order to understand the behavior of the system; estimation assumes that the model is at least approximately *correct* – that is, well specified – to make inference about the true rwDGP. (Richiardi, 2018c, pp. 184-185)

Second, as ABMs generally involve many parameters and non-linearities, it can be very burdensome and empirically unmanageable to estimate them. This has so far deterred estimation of ABMs, and harmed the diffusion of the methodology. The good news is that the development of computational techniques and the increasing availability of computer power have made the issue less untreatable.

In the last two sections, we presented some of the main calibration techniques used in ABMs. Now, we will focus on the main estimation methods available so far.

3.3.6.1. Simulation-Based Estimation

Simulation-based methods have been introduced back in the 1990s, following the developments of computer power. According to Richiardi (2018c, pp. 184-185):

> The basic idea with simulation-based econometrics is to replace the evaluation of analytical expressions about theoretical (model) quantities with their numerical counterparts computed on the simulated data. The (simulated) theoretical quantities, which are functions of the parameters to be estimated, can then be compared with those computed on the real on the real (observed) data as in any estimation procedure. If the model is correctly specified – and some technical conditions hold – for large samples, the observed quantities tend to the theoretical quantities, at the 'true' values of the parameters. Because the simulated quantities also tend to the theoretical quantities, the observed quantities converge to the simulated quantities.

Two families of approaches can be followed when performing simulated-based estimation. The *first* follows a *frequentist approach*, in which the procedure is, roughly speaking, to look at the values of the parameters that minimize the distance between the simulated and the observed quantities. This procedure is known in general as *Simulated Minimum Distance (SMD).* In this general class, the most common techniques are: *the method of simulated moments (MSM), indirect inference (II), and simulated maximum likelihood (SML).*

The task of comparing real and artificial data requires the computation of some statistics y, both in the real and in the artificial data, and then the aggregation into a unique measure of distance. These statistics are computed just once in the real data (which do not change) and once every iteration until convergence in the real data, which depends on the value of the structural parameters. The change in the value of the parameters of each iteration is proceeded following some optimization algorithm, with the goal of minimizing the distance.

The *second* family of approach is the Bayesian. According to Richiardi (2018c, p. 191):

> In Bayesian analysis, one starts with a prior knowledge (sometimes imprecise) expressed as a distribution on the parameter space and updates this knowledge according to the posterior distribution given the data. Classical Bayesians still believe in an unknown 'true' model, as in the frequentist approach. However, rather than aiming at identifying the 'true' values of parameters (or a corresponding confidence interval), they use the information contained in the data to update the subjective beliefs about them. On the other hand, subjective Bayesians do not believe in such true models and think only in terms of predictive distribution of a future observation.
> For frequentists (and classical Bayesians), parameters are assumed to be fixed (at least within a group or condition) and inference is based on the sample space of hypothetical outcomes that might be observed by replicating the experiment many times. For subjective Bayesians, on the other hand parameters are treated as random quantities, along with the data, and inference is based on posterior distributions.

In the following sections, we will present the general ideas behind these approaches.

3.3.6.1.1. The Method of Simulated Moments (MSM)

This method proposes a solution to properly characterize both the model and the real data that considers the *longitudinal means* of the selected statistics. Rather than seeking consistency in *sample size*, consistency in *time* is achieved. By increasing the length of the observation period, both for the real and the simulated data, the estimates become more precise and converge toward the true value of the parameters.

Broadly speaking, in the MSM, different orders of moments of the time series of interest are used, and then weighted to take into account their uncertainty. The idea behind this methodology is to allow parameters estimated with a higher degree of uncertainty to count less in the final measure of distance between the real and the artificial data. It is important to note that having different weights affects the efficiency of the estimates, but not their consistency.

The moment estimator is:

$$\hat{\theta} = \arg\min_{\theta}[\mu^*(\theta) - \mu_R]'\, W^{-1}[\mu^*(\theta) - \mu_R] \tag{123}$$

where W is a positive definite matrix of weights, $\mu^*(\theta)$ is the simulated moment, and μ_R is the real moment.

The model analysis proceeds as follows. If the number of moments is equal to the number of structural parameters to be estimated, the model is *just-identified*. The minimized distance, for the estimated values of the parameters, is therefore 0 in the limit (as the sample size increases), supposing the model is correctly specified. If the number of moments is higher than the number of parameters, the model is *over-identified* and the minimized distance is always positive. If it is lower, the model is *under-identified*.

3.3.6.1.2. Indirect Inference (II)

In the II method, the basic idea is to use the coefficients of an *auxiliary model*, estimated both on the real and on the simulated data, to describe the data, that is, to use the coefficients as summary statistics on the original model. The model prescribes the following steps:

(i) Simulate the model for a candidate parameters vector θ_i and obtain artificial data;

(ii) Estimate the parameters β of a (possibly misspecified) auxiliary model $y_t = f(\beta, z_t)$, where z_t are the explanatory variables;

(iii) Change the structural parameters θ of the original model until the distance between the estimates of the auxiliary model using real and artificial data is minimized, as follows:

$$\hat{\theta} = \arg\min_{\theta}[\hat{\beta}(\theta) - \hat{\beta}_R]' \, W^{-1}[\hat{\beta}(\theta) - \hat{\beta}_R] \tag{124}$$

where W is a positive definite matrix of weights; $\hat{\beta}(\theta)$ is the estimated parameter of the auxiliary model; and $\hat{\beta}_R$ is the estimated parameter of the original model.

The logic is the same of the simulated moments. If the number of the parameters of the auxiliary model is equal to the number of parameters in the original model, the original model is *just-identified*, and the distance between the estimated coefficients on the real and on the simulated data (if the model is correctly specified) goes in the limit to zero. If the number of parameters in the auxiliary model is bigger than the number of parameters in the original model, the original model is *over-identified*, and the distance between the estimated coefficients remains positive. Finally, if the number of parameters in the auxiliary model is smaller than the number of parameters in the original model, the original model is *under-identified*.

3.3.6.2. Bayesian Estimation

The fundamental equation for Bayesian methods is the Bayes theorem:

$$p(\theta|Y^R) \propto \mathcal{L}(\theta; Y^R)p(\theta) \tag{125}$$

where $p(\theta)$ is the prior distribution of the parameters, $\mathcal{L}(\theta; Y^R) \equiv p(Y^R|\theta)$ is the likelihood of observing the data $Y^R \equiv \{y_t^R\}$ (with $t = 1, \dots, T$) given the value of the parameters, and $p(\theta|Y^R)$ is the posterior distribution (i. e., the updated distribution once the information coming from the data is properly considered).

As an input to the simulation process, Bayesians use the prior distribution of the parameters and get back a posterior distribution. In the process, knowledge gets updated by the information contained in the data. The prior distribution typically comes from other studies or subjective evaluations. A uniform distribution in the allowed range of parameters is often used as a way to introduce uninformative priors. All in all, the prior is a distribution, which through application of the Bayes theorem yields another distribution as an output.

We can point out three differences between the Bayesian approach and the SMD methods. First, in the Bayesian approach, there is no maximization involved. Second, rather than obtaining a point estimative for the parameters, we get a distribution. Third, prior knowledge may be incorporated.

Sampling the posterior distribution $p(\theta|Y^R)$ involves two computationally intensive processes. The first obtain an estimate for the likelihood $\mathcal{L}$, given the values of θ. The second involves the iteration over different values of θ.

The estimation of the likelihood, that is, the probability of observing the data, given the current values of the parameters, can be done when it is not feasible to analytically derivate it. The process is done when it is repeatedly sampling from the model output.[51]

Once the likelihood is known, the application of the Bayes theorem allows the model to get a probability density function for the posterior distribution, at one given value of θ. However, to recover the whole shape of the posterior distribution, many values need to be sampled.

There are four main classes of efficient sampling schemes to obtain samples from a function of θ: *the rejection sampling*, *the importance sampling*, *the Markov chain Monte Carlo*, and *the sequential Monte Caro methods.*[52]

In the last fifteen years, a new set of methods have appeared to produce approximations of the posterior distributions without relying on the likelihood. These methods are labelled *likelihood-free methods*. The best-known class is the *Approximate Bayesian Computation (ABC).* In what follows, we give an overview of this method.

3.3.6.2.1. Approximate Bayesian Computation (ABC)

In standard Bayesian methods, the likelihood function provides the fit of the model with the data. However, the likelihood is often computationally impractical to

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[51] See Richiardi (2018c, pp. 211-214) for a detailed explanation.

[52] It is beyond the scope of this work to detail how they work. For an excellent survey on this subject, see Hartig et al. (2011).

evaluate. The basic idea of the ABC is to replace the evaluation of the likelihood with a 0-1 indicator, describing whether the outcome of the model is close enough to the observed data.

To perform such a task, a few procedures need to be done. First, the model outcome and the data must be summarized. Then, a distance between the simulated and the real data is computed. The model is considered close enough to the data if the distance falls within the admitted tolerance,

Taking it to a properly formalism, the basic ABC works as follows:

(i) A candidate vector θ_i is drawn from a prior distribution;

(ii) A simulation is done with parameters vector θ_i, obtaining simulated data from the model density $p(y|\theta_i)$;

(iii) The candidate vector is ether retained or dismissed depending on whether the distance between the summary statistics computed on the artificial data $S(y(\theta))$ and the summary statistics computed on the real data $S(y_R)$ is within or outside the admitted tolerance $h : d(S, S_R) \leq h$.

This procedure is repeated N times. The retained values of the parameters define an empirical approximated posterior distribution.

As we can notice, there are three main ingredients in ABC: (i) the section of *summary statistics*; (ii) the definition of a *distance measure*; and (iii) the definition of a *tolerance threshold*. The most challenging choice concerns the first. The standard scheme to select to select summary statistics for ABC is the *rejection sampling* (i. e., candidates are drawn from the prior distribution, and only those who perform well are maintained). However, as Richiardi (2018c) argues, this is not very efficient, mainly if the prior distribution differs significantly from the posterior.

In fact, this topic is an active area of research. In recent years, we have seen the development of techniques to provide guidance in the selection of the summary statistics (see Fearnhead and Prangle, 2012), as well as the use of ABC with more efficient sampling schemes (see Sisson et al., 2016).

In summary, the main difference between ABMs estimation and more standard methods lies in the higher computational complexity of ABMs. Often, likelihood-based methods are impractical, unless very few parameters are involved. This challenging task of empirical validating ABMs has been so far restricted to a few and relatively simple cases. Surely, this is going to change as the field of agent-based modelling becomes more and more mature. As a matter of fact, likelihood-free methods (e. g. ABC) seem therefore promising, especially when coupled with the use of efficient Monte Carlo sampling (Richiardi, 2018c; Sisson et al., 2016).

4. CONCLUSIONS

The recent crisis of 2007-2008 has exposed some weaknesses in the standard macroeconomic modelling approach, grounded upon DSGE models. The main criticisms raised against these models are related to their limitations to forecast the occurrence of large-scale economic turmoils and therefore to provide effective policy advices. Despite their undoubtedly fruitful advances over earlier macroeconomic models, the DSGE framework fails when dealing with macroeconomics' complexity. In this sense, ABMs can be a valid and flexible tool for studying macroeconomics - here understood as a complex social system with many dynamic and interacting components.

This work defended the idea that a more productive research agenda for macroeconomic modelling should avoid any the insular behavior and benefit from a combination of both modelling approaches. In fact, the call for a joint contribution of different approaches is shared by some of the most important macroeconomists in the world. Lindé (2018) makes it clear: "I believe that other models can be important complements and sometimes even substitutes to DSGEs, depending on the question addressed and resources available for modelling and maintenance" (Lindé, 2018, p. 271).

In our view, a possible way of accomplishing such a task starts with a systematically comparison of the two modelling frameworks in order to make them communicate with each other and to yield better policy analyses. But it is not a trivial procedure.

Given the general theoretical formalism of both the DSGE and the ABM's structure, one can notice why it is so difficult to compare and combine results between them. As shown, the theoretical architecture of ABMs is defined in terms of a set of coupled difference equations describing the evolution over time of the state variables characterizing each agent. Couplings come from interactions of one sort or another. In general, due to its high dimensionality the system cannot be solved analytically, and the conditions for an exact aggregation conducive to a representative agent are not respected. The model therefore must be simulated at the computer. In other words, the modeller asks a computer program to solve the system for him in a *specific* case, i.e. for a specific combination of parameter values and initial conditions.

Therefore, before entering the simulation stage, the main modeling problem is the choice of parameter values and of initial conditions for state variables and populations' size and attributes. Such a choice is not independent from the empirical validation of the model, that is, the capability of the model to reproduce some chosen stylized facts, both at the micro and at the macro level. Although parameterization is frequently guided by little else than this, it was shown some of the most important advances on this subject. Of course, the selection of the empirical evidence used for comparison is crucial, as it amounts to defining the criteria against which the model is evaluated. Historical behavior itself passes through a process of analysis and simplification that leads to the identification of a set of s*tylized facts*, which are generally defined in probabilistic terms. In the end, therefore, the model is evaluated according to the extent it is able to statistically replicate a set of selected stylized facts.

At the micro level, the main goal of any validation exercise is to assess the capability of the model to replicate some stylized facts concerning statistical distributions of individual-level state variables (e. g. the right skewed distribution of firms' size or of the income distribution). At the macro level, the main goal of validation is to assess whether the model is able to generate (by means of bottom-up simulation procedures) statistical aggregates which replicate some stylized facts concerning aggregate variables, such as GDP, aggregate unemployment or inflation. Sensible initial choices of parameters, guided mainly by reasonable approximations to well-known stylized facts, should allow the model to replicate satisfactorily those empirical regularities. Once a satisfying initialization choice has been defined, Monte Carlo simulations can be run to check for the robustness of results as the parameter space is suitably explored (Delli Gatti et al. 2011).

On the other hand, DSGE models are simpler, analytically resolvable (albeit many times unmanageable) and may be flexible enough to be used for different purposes. The baseline procedure - according to which the aggregate consumer expenditure is modeled as the solution to the Euler equation (which is a condition for intertemporal optimality) of a representative household, under the hypothesis of rational expectations – has logical coherence and can be proved mathematically, although the assumptions used are sometimes too far from the reality.

It is easy to notice the alleged differences in terms of *mathematical robustness* between analytical models (DSGE models) and computer-simulated models (ABMs). In analytical models, the behavioral rules typically have a simple structure, with either limited or global interaction, and heterogeneity is kept to a minimum. Functional forms are often linear (or log-linearized). Aggregation is performed on selected variables by taking expectations over the stochastic terms, which are conveniently specified.

DSGE models may exaggerate individual rationality and foresight, and understate the importance of heterogeneity, that is, differences between agents focusing mostly on the way economic agents interact through aggregate prices. The AB models offer a more flexible approach to the role of other social interactions between individual agents in the economy by defining the characteristics and behavior of individual heterogeneous agents with limited rationality, information and foresight. On the other hand, AB models may exaggerate errors in individual decision-making, since they usually model only simple strategies that are far from optimal choices and that evolve in time.

In ABMs, little restrictions are made on the specification of the behavioral rules, but this freedom comes at a price. The problem is that agents can depart from rationality in an infinite number of ways leading into what some economists refer to as a *wilderness*. Moreover, the equation for the macro dynamics (see equation 118) can easily grow enormous, hindering any attempt at symbolic manipulation. Another issue must be highlighted. It refers to the connection between inputs and outputs done by the IOT function. As Richiardi (2018b, p. 42) say:

> […] the connection between inputs and outputs is the IOT function, the black box through which only *inductive* evidence based on simulated data can be obtained. The proof of the results thus lies in the code, rather than in mathematical reasoning as in analytical models. This is why it is fundamental, in ABMs, to write the simulation code in a clear and transparent way, document it and make it public. Also, supporting evidence for the working of the 'black box', the shape of the inferred IOT function should be provided, either in terms of analytical results for simple cases or in terms of intuition explaining why the simulated results are obtained.

In summary, despite the differences and weaknesses of both the ABMs and the DSGE models, this work embraced the idea that exploiting diversity in macroeconomic modelling by combining DSGE and ABMs may improve our ability to deal with complexity in economics and yield a more diversified and productive approach for the field. In our view, the first step into this direction was given. Of course, it opens a wide window of opportunity for new researches, such as a proper methodology to compare and combine ABMs and DSGE models. If the resulting approach will tend more to DSGE or to ABMs only time will say. More important than this is that exploiting diversity in macroeconomic modelling may be beneficial when making sense of the economy and when setting policies to shape the economy. By doing so, macroeconomic policy recommendations may avoid the dangerous issue of being prisoner of a single outlook. Should this work contribute a few steps along this promising journey, our mission would be accomplished.

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